The 49th Symposium on Ring Theory

and Representation Theory

ABSTRACT

Osaka Prefecture University, Osaka

August 31 – September 3, 2016

Program

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A generalization of dual symmetry and reciprocity for symmetric algebras

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Hirotaka Koga (Tokyo Denki University)
On modules of infinite reduced grade

10:45–11:15 Naoya Hiramatsu (National Institute of Technology, Kure College)
On the relations for Grothendieck groups of Cohen-Macaulay modules over Gorenstein rings

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Topoloical data analysis and quiver representation I

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Introduction to quiver varieties I

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9:30–10:00 Michio Yoshiwaki (Shizuoka University/Osaka City University)
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14:20–14:50 Ayako Itaba (Shizuoka University)
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Tilting and cluster tilting associated with reduced expressions in Coxeter groups

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On mutation of τ-tilting modules

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14:20–14:50 Osamu Iyama (Nagoya University)
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Classifying dense subcategories of exact categories via Grothendieck groups

16:20–16:50 Ryo Ohkawa (Kyoto University)
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17:00–17:50 Helmut Lenzing（Universität Paderborn）
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September 3 (Saturday)

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10:15–10:45 Laurent Demonet（Nagoya University）
Algebras of partial triangulations

10:45–11:15 Tsutomu Nakamura (Okayama University)
Local duality principle and Grothendieck's vanishing theorem

11:30–12:00 Sota Asai (Nagoya University)
Bricks and 2-term simple-minded collections

The structure of the Sally module of integrally closed ideals

Kazuho Ozeki

This talk is based on a joint work with Maria Evelina Rossi ([1]).

The first two Hilbert coefficients of a primary ideal play an important role in commutative algebra. In this talk we give a complete structure of the Sally module of integrally closed $\mathfrak{m}$-primary ideals I in a Cohen-Macaulay local ring $(A, \mathfrak{m})$ satisfying the equality $\mathrm{e}_1(I) = \mathrm{e}_0(I) - \ell_A(A/I) + \ell_A(I^2/QI) + 1$, where Q is a minimal reduction of I, and $\mathrm{e}_0(I)$ and $\mathrm{e}_1(I)$ denote the first two Hilbert coefficients of I.

Let, for an $\mathfrak{m}$-primry ideal in A and a minimal reduction Q of I,

$$R = \mathrm{R}(I) := \oplus_{n \geq 0} I^n, \ T = \mathrm{R}(Q) := \oplus_{n \geq 0} Q^n \quad \text{and} \quad G = \mathrm{G}(I) := \oplus_{n \geq 0} I^n/I^{n+1}$$

respectively denote the Rees algebras of I, Q, and the associated graded ring of I.

We set, for each $i \geq 1$,

$$C^{(i)} = (I^i R/I^i T)(-i+1)$$

and let $L^{(i)} = T[C^{(i)}]_i$. Then the natural exact sequences $0 \to L^{(i)} \to C^{(i)} \to C^{(i+1)} \to 0$ of graded T-modules hold true for all $i \geq 1$ ([3]). We notice here that $C^{(1)} = IR/IT$ is called the Sally module of I with respect to Q ([2]).

In this talk, we set $C = C^{(2)} = (I^2R/I^2T)(-1)$ and we shall explore the structure of the graded module C. The main result of this talk is stated as follows, where $B = T/\mathfrak{m}T \cong (A/\mathfrak{m})[X_1, X_2, \cdots, X_d]$ denotes the polynomil ring over $A/\mathfrak{m}$.

Theorem 1. *Suppose the I is an integrally closed $\mathfrak{m}$-primary ideal in A. Then the following conditions are equivalent:*

(1) $\mathrm{e}_1(I) = \mathrm{e}_0(I) - \ell_A(A/I) + \ell_A(I^2/QI) + 1$,

(2) $C \cong (X_1, X_2, \cdots, X_c)B(-1)$ *as graded T-modules for some $1 \leq c \leq d$.*

When this is the case, $c = \ell_A(I^3/QI^2)$, $I^4 = QI^3$, and we also have the following:

(i) $\operatorname{depth} G \geq d - c$, *and* $\operatorname{depth} G = d - c$, *if $c \geq 2$.*

(ii) *Suppose $c = 1 < d$. Then we have*

$$\mathrm{e}_i(I) = \begin{cases} \mathrm{e}_1(I) - \mathrm{e}_0(I) + \ell_A(A/I) + 1 & \text{if } i = 2, \\ 1 & \text{if } i = 3 \text{ and } d \geq 3, \\ 0 & \text{if } 4 \leq i \leq d. \end{cases}$$

(iii) *Suppose $2 \leq c < d$. Then we have*

$$\mathrm{e}_i(I) = \begin{cases} \mathrm{e}_1(I) - \mathrm{e}_0(I) + \ell_A(A/I) & \text{if } i = 2, \\ 0 & \text{if } i \neq c+1, c+2, \ \ 3 \leq i \leq d, \\ (-1)^{c+1} & \text{if } i = c+1, c+2, \ \ 3 \leq i \leq d. \end{cases}$$

(iv) *Suppose $c = d$. Then we have*

$$\mathrm{e}_i(I) = \begin{cases} \mathrm{e}_1(I) - \mathrm{e}_0(I) + \ell_A(A/I) & \text{if } i = 2 \text{ and } d \geq 2, \\ 0 & \text{if } 3 \leq i \leq d. \end{cases}$$

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A generalization of dual symmetry and reciprocity for symmetric algebras

Taro Sakurai

Slicing a module into semisimple ones is useful to study modules. Loewy structures provide a means of doing so. To establish the Loewy structures of projective modules over a finite dimensional symmetric algebra over a field F, the Landrock lemma [1] is a primary tool. The lemma and its corollary relate radical layers of projective indecomposable modules P to radical layers of the F-duals P^* ("dual symmetry") and to socle layers of P ("reciprocity").

In this talk, we explain a generalization of these results to an *arbitrary* finite dimensional algebra A. Our main theorem below, which is the same as the Landrock lemma for finite dimensional *symmetric* algebras, relates radical layers of projective indecomposable modules P to radical layers of the A-duals $P^\vee$ and to socle layers of injective indecomposable modules νP where ν is the Nakayama functor. A key tool to prove the main theorem is a pair of adjoint functors, which we call socle functors and capital functors.

Theorem 1 (see [2, Theorem 1.3])**.** *For a finite dimensional algebra A over a field F, let P_i and P_j be the projective covers of simple A-modules S_i and S_j respectively. For an integer $n \geq 1$ the nth radical layer and the nth socle layer are denoted by rad_n and soc_n respectively. Then we have F-linear isomorphisms*

$$\operatorname{Hom}_A(\operatorname{rad}_n P_i, S_j) \cong \operatorname{Hom}_{A^{\mathrm{op}}}(\operatorname{rad}_n(P_j^\vee), S_i^*)$$

and

$$\operatorname{Hom}_A(\operatorname{rad}_n P_i, S_j) \cong \operatorname{Hom}_A(S_i, \operatorname{soc}_n \nu P_j).$$

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On modules of infinite reduced grade

Mitsuo Hoshino, Noritsugu Kameyama and Hirotaka Koga

This talk is besed on [2]. Let R, A be right Noetherian rings and V an (A, R)-bimodule. Our aim is to provide a sufficient condition on V which enables A to inherit from R certain homological properties. Especially, we will show that if the generalized Nakayama conjecture is true for R then so is for A.

We denote by Mod-R the category of right R-modules, by mod-R the full subcategory of Mod-R consisting of finitely presented modules and by $\mathcal{P}_R$ the full subcategory of mod-R consisting of projective modules. Let $\mathcal{G}_R$ denote the full subcategory of mod-R consisting of $X \in$ mod-R with $\mathrm{Ext}^i_R(X, R) = 0$ for all $i \geq 1$ and, for convenience's sake, set $\mathcal{G}^0_R = \{X \in \mathcal{G}_R \mid \mathrm{Hom}_R(X, R) = 0\}$. We denote by R^{op} the opposite ring of R and consider left R-modules as right R^{op}-modules. Let $\{S_\lambda\}_{\lambda \in \Lambda}$ be a complete set of non-isomorphic simple modules in Mod-R^{op}. For each $\lambda \in \Lambda$ we set $E_\lambda = E_{R^{\mathrm{op}}}(S_\lambda)$, the injective envelope of S_λ in Mod-R^{op}.

Assume that V satisfies the following three conditions: (a) $V_R \in \mathcal{G}_R$; (b) ${}_AV$ is faithfully flat; and (c) $\mathrm{inj}\ \mathrm{dim}_A V \otimes_R E_\lambda < \infty$ for all $\lambda \in \Lambda$. Then we will show that if $\mathcal{G}^0_R = \{0\}$ then $\mathcal{G}^0_A = \{0\}$, and that if $\mathcal{G}_R$ consists only of torsionless modules then so does $\mathcal{G}_A$. It should be noted that if A is a Frobenius extension of R and $V = A$ then the conditions above are satisfied.

Next, assume further that for any maximal right ideal $\mathfrak{m}$ in A, setting $\mathfrak{A} = \{x \in R \mid Vx \subset \mathfrak{m}V\}$, $R/\mathfrak{A}$ is a semisimple ring. We will show that if the generalized Nakayama conjecture is true for R then so is for A.

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Relations for Grothendieck groups of Cohen-Macaulay modules over Gorenstein rings

Naoya Hiramatsu

Let $(R, \mathfrak{m})$ be a commutative Cohen-Macaulay complete ring. We denote by $\mathrm{mod}(R)$ the category of finitely generated R-modules with R-homomorphisms and by $\mathcal{C}$ the full subcategory of $\mathrm{mod}(R)$ consisting of all Cohen-Macaulay R-modules. Set $\mathrm{G}(\mathcal{C}) = \bigoplus_{X \in \mathrm{ind}\mathcal{C}} \mathbb{Z}\cdot[X]$, which is a free abelian group generated by isomorphism classes of indecomposable objects in $\mathcal{C}$. We denote by $\mathrm{EX}(\mathcal{C})$ a subgroup of $\mathrm{G}(\mathcal{C})$ generated by

$$\{[X] + [Z] - [Y] \mid \text{there is an exact sequence } 0 \to Z \to Y \to X \to 0 \text{ in } \mathcal{C}\}.$$

We also denote by $\mathrm{AR}(\mathcal{C})$ a subgroup of $\mathrm{G}(\mathcal{C})$ generated by

$$\{[X] + [Z] - [Y] \mid \text{there is an AR sequence } 0 \to Z \to Y \to X \to 0 \text{ in } \mathcal{C}\}.$$

Let $K_0(\mathcal{C})$ be a Grothendieck group of $\mathcal{C}$. By the definition, $K_0(\mathcal{C}) = \mathrm{G}(\mathcal{C})/\mathrm{EX}(\mathcal{C})$.

On the relation for Grothendieck groups, Butler[3], Auslander-Reiten[2], and Yoshino[5] prove the following theorem.

Theorem 1. [3, 2, 5] *If R is of finite representation type then* $\mathrm{EX}(\mathcal{C}) = \mathrm{AR}(\mathcal{C})$.

Here we say that R is of finite representation type if there are only a finite number of isomorphism classes of indecomposable Cohen-Macaulay R-modules.

Auslander conjectured the converse of Theorem 1 is true. Actually it has been proved by Auslander[1] for Artin algebras and by Auslander-Reiten[2] for complete one dimensional domain. In this talk we consider for the case of complete Gorenstein local rings with an isolated singularity.

Theorem 2. [4] *Let R be a complete Gorenstein local ring with an isolated singularity and with algebraically closed residue field. If* $\mathrm{EX}(\mathcal{C}) = \mathrm{AR}(\mathcal{C})$*, then R is of finite representation type.*

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Representations of quivers and Grothendieck derivators I, II

Jan Stovicek

The theory of derivators goes back to Alex Heller [8], Alexander Grothendieck [2], Jens Franke [1], and others. It provides a relatively elementary axiomatic framework which fixes some deficiencies of triangulated categories (for example, the non-functoriality of the cone construction).

The main idea behind derivators is that, starting with an abelian category or with a model category, one considers not only the corresponding derived or homotopy category alone, but rather simultaneously derived categories or homotopy categories of various diagram categories. This is where insights from representation theory start to be very useful since (derived or homotopy) categories of *representations* of small categories in the original abelian or model category are considered.

In the talks this theory will be explained, based on the introductory text [3], and the series of papers [4], [5], [6], and [7].

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Canonical bimodules of Morita algebras

Kunio Yamagata

This is a part of joint work with Ming Fang and Otto Kerner.

- All algebras are finite dimensional over a field K, and modules are finite dimensional left modules,
- A^{op} is the opposite algebra of an algebra A,
- $D = \mathrm{Hom}_K(-, K)$ the standard duality.

Let A be a finite dimensional algebra over a field K. The A-bimodule

$$V = \mathrm{Hom}_A(D(A), A)$$

is called the *canonical bimodule* of A, and A is said to be a *Morita algebra* (over a selfinjective algebra B) if it satisfies the following equivalent conditions:

(i) A is isomorphic to the endomorphism algebra of a generator over a selfinjective algebra B.
(ii) $A \cong \mathrm{End}_{A^{op}}(V)$ canonically.
(iii) $A \cong \mathrm{End}_A(V)^{op}$ canonically.

The A-bimodules $\mathrm{Ext}^i_A(D(A), A)$, $i > 0$, play an essential role in representation theory of preprojective algebras by Ringel and Keller-Iyama. The case $i = 0$, that is, the A-bimodule $V = \mathrm{Hom}_A(D(A), A)$, is still strongly connected to selfinjective algebras. An important aspect of V was first pointed out by M. Fang - S. Koenig (2011) in their study of *gendo-symmetric* algebras (= Morita algebras over symmetric algebras), and then another feature of V was found by O. Kerner - K. Yamagata (2013) for arbitrary Morita algebras, see the definition of Morita algebras. In this talk I will report further results on the canonical bimodules from a joint work with Fang and Kerner. One of the main results is: *an algebra A has the dominant dimension greater than or equal to two if and only if*

$$D(A) \otimes_A V \otimes_A D(A) \cong D(A)$$

as A-bimodules, which will be applied to get a new characterization of Morita algebras.

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The Krull dimension of power series rings

Jung Wook Lim (Department of Mathematics, Kyungpook National University, Republic of Korea)

In this talk, we investigate to study the calculation of the Krull dimension of power series rings over nonNoetherian domains.

New results on piecewise prime rings

Gangyong Lee* and S. Tariq Rizvi
(Sungkyunkwan University*, The Ohio State University)

The study of prime rings and prime ideals has been an important topic of study in Ring Theory because these notions help provide the description of structures of rings. As the class of piecewise prime rings is one of the special class of quasi-Baer rings, the piecewise prime rings have a general triangular matrix representation with prime rings on the diagonal. A quasi-Baer ring is said to be *piecewise prime* (PWP) if the ring has a complete set of triangulating idempotents. Note that the class of prime rings is also that of quasi-Baer rings

The notion of PWP rings was introduced by Birkenmeier-Heatherly-Kim-Park in 2000. Although it is known that the corner ring of a PWP ring is also a PWP ring when the idempotent is a right (left) semicentral idempotent or full idempotent, whether we do not know that it holds true for a general idempotent, until now. In this talk, after we briefly provided the background of PWP rings, we show that every corner ring of a PWP ring is a PWP ring. Also, it is shown that the column (and row) finite matrix ring over a PWP ring is a PWP ring. This talk is based on a joint work with S. Tariq Rizvi.

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Topological data analysis and quiver representation

Yasuaki Hiraoka

In this talk, recent progresses on topological data analysis and persistent homology are presented. In particular, I focus on several connections of persistent homology to commutative algebra and quiver representations [1]. Furthermore, I demonstrate several applications using persistent homology in materials science [2], and show further mathematical problems in representation theory motivated from these applications.

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INTRODUCTION TO QUIVER VARIETIES

Hiraku Nakajima

Let $Q = (Q_0, Q_1)$ be a finite quiver. We consider the doubled quiver, which is obtained by adding arrows in the opposite directions to Q. Let $\overline{Q_1}$ denote the set of opposite arrows. We denote the *incoming* and *outgoing* vertices of an arrow h by $i(h)$ and $o(h)$ respectively.

In 1994, I introduced *quiver varieties* $\mathfrak{M} \equiv \mathfrak{M}(V,W)$ as moduli spaces of (framed) representations of the preprojective algebra of a quiver $Q = (Q_0, Q_1)$ [2]:

$$\mathbf{M}(V,W) = \bigoplus_{h\in Q_1\sqcup\overline{Q_1}} \operatorname{Hom}(V_{o(h)}, V_{i(h)}) \oplus \bigoplus_{i\in Q_0} \operatorname{Hom}(W_i, V_i) \oplus \operatorname{Hom}(V_i, W_i),$$

$$\mu\colon \mathbf{M}(V,W) \to \bigoplus_i \mathfrak{gl}(V_i); \quad \mu(B,I,J)_i = \sum_{\substack{h\in Q_1\sqcup\overline{Q_1} \\ i(h)=i}} \varepsilon(h) B_h B_{\overline{h}} + I_i J_i,$$

$$\mathfrak{M}(V,W) = \mu^{-1}(0)/\prod_i \mathrm{GL}(V_i),$$

where $\varepsilon(h) = 1$ if $h \in Q_1$, -1 if $h \in \overline{Q_1}$. The quotient $\mu^{-1}(0)/\prod_i \mathrm{GL}(V_i)$ is defined carefully, using the geometric invariant theory in algebraic geometry, but let us omit the detail at this moment. We consider all varieties over $\mathbb{C}$.

My motivation was *not* to study representation theory of the preprojective algebra, rather study of structures of quiver varieties, such as symplectic geometry, topology, etc, as I was a geometer, *not* a representation theorist.

Let $\mathfrak{g} = \mathfrak{g}_Q$ be the Kac-Moody Lie algebra corresponding to Q. Namely we assume Q has no edge loops, and consider the underlying graph of Q by forgetting the orientation of Q. Then consider it as a Dynkin diagram, and associate a Kac-Moody Lie algebra.

Let us fix W, and consider the direct sum of middle degree (topological) homology groups of $\mathfrak{M}(V,W)$ for various V (dimension vectors):

$$\bigoplus_V H_{d(V,W)}(\mathfrak{M}(V,W), \mathbb{C}), \qquad (d(V,W) = \dim \mathfrak{M}(V,W)).$$

Then it has a structure of an irreducible integrable highest weight representation of the Kac-Moody Lie algebra $\mathfrak{g}$, with the highest weight given by $\sum_{i\in Q_0} \dim W_i \cdot \Lambda_i$.

This result was motivated by earlier results by Ringel [4] and Lusztig [1] constructing the upper triangular subalgebra $\mathbf{U}^-$ of the quantized enveloping algebra $\mathbf{U} = \mathbf{U}_q(\mathfrak{g})$ and its canonical base. (In the earlier paper [2], we consider the space of constructible functions instead of the homology group.

Thus representation theories of two different (Lie) algebras, the preprojective algebra and the Kac-Moody Lie algebra, are linked through geometry.

The purpose of my lectures is to explain this result, as well as other related results.

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On Loewy lengths of centers of blocks

Yoshihiro Otokita

Let G be a finite group and F an algebraically closed field of prime characteristic $p > 0$. For each block ideal B of the group algebra FG we can define the defect number d_B. The invariant d_B is a non-negative integer and related to the structure of B. For example blocks with $d_B = 0$ or 1 are well known (see Nagao-Tsushima [3, Theorem 6.37] and Linckelmann [2]).

Here we denote by $\mathrm{ll}B$ and $\mathrm{ll}\mathbf{Z}B$ the Loewy lengths of B and its center $\mathbf{Z}B$, respectively, and deals with the problem of classifying blocks by them.

Some studies have determined all blocks with $\mathrm{ll}B \leq 3$ (see Okuyama [5]). Moreover, recent papers Koshitani-Külshammer-Sambale [1] and Sambale [7] investigate some cases for $\mathrm{ll}B = 4$.

In this talk we focus on $\mathrm{ll}\mathbf{Z}B$. Okuyama [4] has proved that $\mathrm{ll}\mathbf{Z}B \leq p^{d_B}$ with equality if and only if B is isomorphic to a matrix ring of a group algebra $F[\mathbb{Z}_{p^{d_B}}]$ where $\mathbb{Z}_{p^{d_B}}$ is a cyclic group of order p^{d_B}. On the basis of this fact, we consider blocks with $p^{d_B} - 3 \leq \mathrm{ll}\mathbf{Z}B \leq p^{d_B} - 1$. Our main theorems indicate that we can classify these blocks into 8 types. These results are based on Otokita [6].

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DECOMPOSITION THEORY OF MODULES: THE CASE OF KRONECKER ALGEBRA

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1. Abstract

Throughout this paper $\Bbbk$ is an algebraically closed field, and all vector spaces, algebras and linear maps are assumed to be finite-dimensional $\Bbbk$-vector spaces, finite-dimensional $\Bbbk$-algebras and $\Bbbk$-linear maps, respectively. Further all modules over an algebra considered here are assumed to be finite-dimensional left modules. For a $\Bbbk$-vector spaces V we denote by $\dim V$ the $\Bbbk$-dimension of V.

Let A be an algebra, $\mathcal{L}$ a complete set of representatives of isoclasses of indecomposable A-modules. Then the Krull-Schmidt theorem states the following. For each A-module M, there exists a unique map $\boldsymbol{d}_M : \mathcal{L} \to \mathbb{N}_0$ such that

$$M \cong \bigoplus_{L \in \mathcal{L}} L^{(\boldsymbol{d}_M(L))}, \tag{1}$$

which is called an *indecomposable decomposition* of M. Therefore, $M \cong N$ if and only if $\boldsymbol{d}_M = \boldsymbol{d}_N$ for all A-modules M and N, i.e., the map $\boldsymbol{d}_M$ is a complete invariant of M under isomorphisms. Note that since M is finite-dimensional, the support $\operatorname{supp}(\boldsymbol{d}_M) := \{L \in \mathcal{L} \mid \boldsymbol{d}_M(L) \neq 0\}$ of $\boldsymbol{d}_M$ is a finite set. We call such a theory a *decomposition theory* that computes the indecomposable decomposition of a module. In the case that $\mathcal{L}$ is already computed, the purpose of this theory is to compute

(1) $\boldsymbol{d}_M$ and
(2) a finite set S_M such that $\operatorname{supp}(\boldsymbol{d}_M) \subseteq S_M \subseteq \mathcal{L}$

for all A-modules M. Note that (2) is needed to give a finite algorithm.

The following is our main result giving a general solution for (1) that extends the well-known solution for Jordan blocks. (2) is solved by using the trace and reject.

Theorem 1. *Let L be an indecomposable A-module and $f : L \to \bigoplus_{X \in J_L} X^{(a(X))}$ with $J_L \subseteq \mathcal{L}$ a source map starting from L. Then we have the following formula:*

$$\boldsymbol{d}_M(L) = \dim \operatorname{Hom}_A(L, M) - \sum_{X \in J_L} a(X) \dim \operatorname{Hom}_A(X, M) + \dim \operatorname{Hom}_A(\tau^{-1}L, M), \tag{2}$$

where $\tau^{-1} := \operatorname{Tr} D : \overline{\operatorname{mod}}\, A \to \underline{\operatorname{mod}}\, A$ is the AR-translation.

Note that this equation always hold because $\tau^{-1}L = 0$ when L is injective. Further the dimensions of Hom spaces can be computed by the ranks of some matrices.

As an example we give an explicit formula of $\boldsymbol{d}_M$ for A-modules M when A is the Kronecker algebra.

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Mutations of splitting maximal modifying modules arising from dimer models

Yusuke Nakajima

In this talk, I will consider a dimer model which is a bipartite graph on the real two-torus. It was introduced in the field of statistical mechanics, and recently string theorists used it for studying quiver gauge theories. Subsequently, relations between dimer models and many branches of mathematics have been discovered. One of the remarkable property is that a good dimer model (which is called consistent) gives a non-commutative crepant resolution (= NCCR) introduced by M. Van den Bergh in [7].

More precisely, we obtain a quiver with potential (Q, W_Q) as the dual of a dimer model. By using such a quiver with potential, we define a certain path algebra with relations called the Jacobian algebra $\mathcal{P}(Q, W_Q)$. Suppose that R is the center of the Jacobian algebra $\mathcal{P}(Q, W_Q)$ arising from a consistent dimer model. Then R is a 3-dimensional Gorenstein toric singularity, and we have a reflexive R-module M such that $\mathcal{P}(Q, W_Q) \cong \mathrm{End}_R(M)$. This algebra is just an NCCR of R [1, 3], that is, it satisfies $\mathrm{gl.dim}\,\mathrm{End}_R(M) < \infty$ and $\mathrm{End}_R(M)$ is a maximal Cohen-Macaulay R-module. Especially this algebra is derived equivalent to the ordinary crepant resolutions of $\mathrm{Spec}\, R$. Also, a reflexive module M satisfying the above condition is called splitting maximal modifying module.

Definition 1. (see [4, 5]) Let $\mathsf{CM}R$ be the category of maximal Cohen-Macaulay R-modules, and $\mathsf{ref}R$ be the category of reflexive R-modules. Then we say $M \in \mathsf{ref}R$ is a *maximal modifying module* (= *MM module*) if $\mathrm{End}_R(M) \in \mathsf{CM}R$, and if there exists $X \in \mathsf{ref}R$ such that $\mathrm{End}_R(M \oplus X) \in \mathsf{CM}R$ then $X \in \mathsf{add}_R M$. Furthermore, we say $M \in \mathsf{ref}R$ is *splitting* if it is a finite direct sum of rank one reflexive modules.

On the other hand, for every 3-dimensional Gorenstein toric singularity R, there exists a consistent dimer model giving an NCCR of R [2, 3]. Therefore, every 3-dimensional Gorenstein toric singularity has an NCCR arising from a consistent dimer model. However, such a dimer model is not unique in general, hence a splitting MM module giving an NCCR is also not unique.

In this talk, I will introduce the notion of the mutation of splitting MM modules to discuss a relationship between splitting MM modules obtained from consistent dimer models. It is a certain operation producing a new splitting MM module from a given one. In particular, I have the following theorem.

Theorem 2 ([6])**.** *Let R be a 3-dimensional complete local Gorenstein toric singularity associated with a "reflexive polygon". Then any two splitting MM R-modules are transformed into each other by repeating the mutation of splitting MM modules.*

Note that the same statement also holds for some special cases, but it is still open for any 3-dimensional Gorenstein toric singularities.

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On representation-finite biserial gendo-symmetric algebras

Aaron Chan

Following [1], an algebra A is a gendo-symmetric algebra if it is isomorphic to the endomorphism ring of a generator over a symmetric algebra B. This is a generalisation of symmetric algebra from the viewpoint of Morita-Tachikawa correspondence [3]. An example of such an algebra is the Auslander algebra of a representation-finite symmetric algebra.

Recall that an algebra is said to be biserial, if the radical of any indecomposable projective module is isomorphic to $U + V$, where U, V are uniserial (have a unique filtration with simple subquotients) and $U \cap V$ is either simple or zero. It is well-known that representation-finite biserial algebras have many nice features - for example one can classify and describe their indecomposable modules via simple combinatorics. Moreover, representation-finite biserial symmetric algebras are precisely the so-called Brauer tree algebras, which are well-known to group representation theorists and undoubtedly the simplest class of symmetric algebras.

It is then natural to consider representation-finite biserial gendo-symmetric algebras, and expect many of its properties can be obtained from simple combinatorics associated to Brauer tree algebras. Indeed, one can show that any representation-finite biserial gendo-symmetric algebra is isomorphic to the endomorphism ring of a generator over a Brauer tree algebra. Moreover, we can classify all possible generators of Brauer tree algebras which gives rise to a representation-finite biserial gendo-symmetric algebra.

It turns out that the indecomposable non-projective direct summands in such a generator are given by maximal uniserial non-projective module or the simple top of a uniserial projective module. In particular, we can use a classical combinatorics - the Green's walk around Brauer tree [2] - to determine the dominant and Gorenstein dimension of these gendo-symmetric algebras.

This is a joint work with René Marczinzik.

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m-Koszul AS-regular algebras and twisted superpotentials

Izuru Mori

This talk is based on a joint work with S. P. Smith [1]. AS-regular algebras is the most important class of algebras to study in noncommutative algebraic geometry. If S is an m-Koszul AS-regular algebra, then it was observed by several people that S is determined by a twisted superpotential. In this talk, we will see that such a twisted superpotential is uniquely determined by S up to non-zero scalar multiples and plays a crucial role in studying S. In particular, we will see in this talk that, using the twisted superpotential w_S associated to S, we can compute:

(1) the Nakayama automorphism of S,
(2) a graded algebra automorphism of S, and
(3) the homological determinant of a graded algebra automorphism of S.

The homological determinant is an essential ingredient for invariant theory of AS-regular algebras. Despite its importance, it is rather mysterious and difficult to compute from the definition, so our result is very useful.

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3-dimensional quadratic Artin-Schelter regular algebras and superpotentials

Ayako Itaba

Let k be an algebraically closed field of characteristic 0, A a graded k-algebra finitely generated in degree 1 and V a k-vector space. First, we recall the definition of Artin-Schelter regular algebras.

Definition 1. ([1]) Let A be a noetherian connected graded k-algebra. A is called a *d-dimensional Artin-Schelter regular* (simply *AS-regular*) *algebra* if A satisfies the following conditions:

(1) $\operatorname{gldim} A = d < \infty$, $\operatorname{Ext}^i_A(k, A) = \begin{cases} k & (i = d), \\ 0 & (i \neq d). \end{cases}$

In this talk, we consider 3-dimensional quadratic AS-regular algebras. These are classified by Artin-Tate-Van den Bergh [2] using a geometric pair (E, σ), where E is a cubic curve of $\mathbb{P}^2$ and σ is an automorphism of E. Also, a 3-dimensional quadratic AS-regular algebra is Koszul, and the quadratic dual $A^!$ of A is a Frobenius algebra. Then, the Nakayama automorphism of $A^!$ is identity if and only if A is a Calabi-Yau algebra ([5]). Now, we give the definition of superpotential.

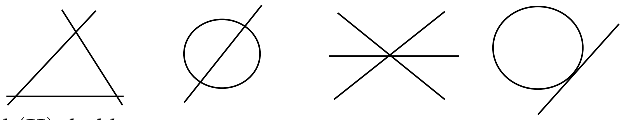
Definition 2. ([3], [4]) For a finite-dimensional k-vector space V, we define the k-linear map ϕ: $V^{\otimes 3} \longrightarrow V^{\otimes 3}$ by $\phi(v_1 \otimes v_2 \otimes v_3) := v_3 \otimes v_1 \otimes v_2$. If $\phi(w) = w$ for $w \in V^{\otimes 3}$, then w is called *superpotential.* Also, for $\tau \in \mathrm{GL}(V)$, we define $w^\tau := (\tau^2 \otimes \tau \otimes \mathrm{id})(w)$, where $\mathrm{GL}(V)$ is the general linear group of V. Moreover, for a subspace W of $V^{\otimes 3}$, we set

- $\partial W := \{(\psi \otimes \mathrm{id}^{\otimes 2})(w) \mid \psi \in V^*,\ w \in W\}$,
- $\mathcal{D}(W) := T(V)/(\partial W)$.

For $w \in V^{\otimes 3}$, $\mathcal{D}(w) := \mathcal{D}(kw)$ is called the *derivation-quotient algebra* of w.

In this talk, our main result is as follows:

Theorem 3. *For the* 3*-dimensional quadratic AS-regular algebra $A = \mathcal{A}(E, \sigma)$ corresponding to E and $\sigma \in \operatorname{Aut} E$, suppose that E is $\mathbb{P}^2$ or the cubic curve of $\mathbb{P}^2$ as follows*:



Then, the following (I) *and* (II) *hold*:

(I): *there exist a superpotential $w \in V^{\otimes 3}$ and an automorphism τ of V such that A and the derivation-quotient algebra $\mathcal{D}(w^\tau)$ of w^τ are isomorphic as graded algebras;*

(II): *there exists a Calabi-Yau AS-regular algebra C such that A and C are graded Morita equivalent.*

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3-dimensional cubic Calabi-Yau algebras and superpotentials

Kenta Ueyama

This talk is based on a joint work [6] with Izuru Mori.

In representation theory of algebras, Calabi-Yau algebras are important class of algebras to study. Since every connected graded Calabi-Yau algebra is AS-regular ([7]), it is interesting to study such algebras from the point of view of both representation theory and noncommutative algebraic geometry.

It was shown that every m-Koszul Calabi-Yau algebra S is isomorphic to a Jacobian algebra $J(\mathsf{w}_S)$ of a unique superpotential w_S up to non-zero scalar multiples ([2], [3], [4]). Moreover, it is known that every 3-dimensional noetherian connected graded Calabi-Yau algebra S generated in degree 1 is either 2-Koszul (quadratic) or 3-Koszul (cubic), so $S \cong J(\mathsf{w}_S)$ for some unique superpotential w_S. Recently, Mori and Smith [4], [5] classified all superpotentials whose Jacobian algebras are 3-dimensional noetherian quadratic Calabi-Yau algebras, and computed the homological determinants of graded algebra automorphisms of 3-dimensional noetherian quadratic Calabi-Yau algebras. As a continuation, in this talk, we focus on studying 3-dimensional noetherian cubic Calabi-Yau algebras.

Let S be a 3-dimensional noetherian Calabi-Yau algebra. If S is cubic, then $\mathsf{w}_S \in V^{\otimes 4}$ where V is a 2-dimensional vector space. First we classify all superpotentials $\mathsf{w} \in V^{\otimes 4}$ such that $J(\mathsf{w})$ are 3-dimensional cubic Calabi-Yau. Using this classification, we obtain the following:

(1) We show that $J(\mathsf{w})$ is 3-dimensional Calabi-Yau except for five algebras up to isomorphisms.
(2) We show that $J(\mathsf{w})$ is 3-dimensional Calabi-Yau if and only if it is a domain as in the quadratic case ([5]).
(3) We compute all possible point schemes (in the sense of Artin, Tate and van den Bergh [1]) for 3-dimensional noetherian cubic Calabi-Yau algebras. By this computation, we see that not all bidegree (2, 2) divisors in $\mathbb{P}^1 \times \mathbb{P}^1$ appear as point schemes. This result contrasts to the fact that all degree 3 divisors in $\mathbb{P}^2$ appear as point schemes of 3-dimensional noetherian quadratic Calabi-Yau algebras ([5]).
(4) We show that if $S = T(V)/(R)$ is a 3-dimensional noetherian cubic Calabi-Yau algebra and σ is a graded algebra automorphism of S, then the homological determinant of σ can be calculated by the formula $\operatorname{hdet} \sigma = (\det \sigma|_V)^2$ with one exception.

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A SPECTRAL ANALYSIS OF NAKAYAMA ALGEBRAS

Helmut Lenzing, Paderborn

The talk deals with joint research with José Antonio de la Peña and partly with Shiquan Ruan. We investigate the class of Nakayama algebras $A_n(r)$ given by a linear quiver with n vertices and zero composition for all r-tuples of adjacent arrows. While their categories of finite dimensional representations are representation-finite and offer no surprises, the attached bounded derived categories, termed *Nakayama categories*, form a rich and interesting domain of research. This is because many Nakayama categories show up in singularity theory. Particular attention will be given to the E-series of (bounded derived categories) formed by the Nakayama categories attached to the algebras $A_3(n)$.

In the focus of my talk will be the mentioned link to singularity theory and a spectral analysis (Coveter transformations, Coveter polynomials, spectral radii) for Nakayama categories. The research complements previous investigations by Happel-Seidel and joint work with Kussin and Meltzer on triangle singularities.

On the Hochschild (co)homology of a monomial algebra given by a cyclic quiver and two zero-relations

Tomohiro Itagaki

This talk is based on [5]. In this talk, we determine the Hochschild (co)homology groups of a monomial algebra over an algebraically closed field given by a cyclic quiver and two zero-relations.

The Hochschild (co)homology of algebras is one of important invariances of derived equivalence. However, in general, it is difficult to determine these algebraic structures. For a monomial algebra over an algebraically closed field, Bardzell [1] gave its minimal projective bimodule resolution. By means of this minimal projective resolution, for some classes of monomial algebras, the module structure and ring structure of the Hochschild cohomology are investigated. However, for a monomial algebra, even the module structure of the Hochschild cohomology is not completely determined.

While, Han [3] gave the Hochschild homology groups of a monomial algebra over a field by means of the Hochschild homology groups of bound quiver algebras given by cyclic subquivers of its ordinary quiver. By the result in [4], for bound quiver algebras of a cyclic quiver, the module structure of the Hochschild homology is given by the Hochschild homology of truncated cycle algebras. In particular, the Hochschild homology of truncated cycle algebras is computed by Han [3] and Sköldberg [7]. However, the dimension formula of the Hochschild homology groups of bound quiver algebras of a cyclic quiver is not known completely.

Let K be an algebraically closed field, $s \geq 3$ a positive integer, Γ_s a cyclic quiver with s vertices and s arrows, and I an admissible ideal of $K\Gamma_s$. The cardinal number of the minimal set of paths in the generating set of I is equal to s if and only if $K\Gamma_s/I$ is a truncated cycle algebra. The Hochschild cohomology groups of a truncated cycle algebra is determined in [2] and [9]. On the other hand, for an algebra $K\Gamma_s/I$ with an ideal I generated by only one path, Xu and Wang [8] investigated its Hochschild homology and cohomology. In this talk, we determine the Hochschild (co)homology groups of $K\Gamma_s/I$, where I is an ideal generated by two paths.

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Tilting and cluster tilting associated with reduced expressions in Coxeter groups

Yuta Kimura

Recently, there are many studies on 2-Calabi-Yau triangulated categories and their cluster tilting objects.

One of well-studied classes of 2-Calabi-Yau triangulated categories was introduced by [3]. Let Q be a finite acyclic quiver and W be the Coxeter group of Q. For each $w \in W$, Buan-Iyama-Reiten-Scott introduced an Iwanaga-Gorenstein algebra $\Pi(w)$. They showed that the stable category $\underline{\mathsf{Sub}}\,\Pi(w)$ is a 2-Calabi-Yau triangulated category, where $\mathsf{Sub}\,\Pi(w)$ is the category of submodules of finitely generated free $\Pi(w)$-modules. They also showed that it has a cluster tilting object $M(\boldsymbol{w})$ associated with a reduced expression $\boldsymbol{w}$ of w.

Another well-studied class of 2-Calabi-Yau triangulated categories is the cluster categories. The cluster category $\mathsf{C}(A)$ of an algebra A of global dimension at most two is introduced by Amiot [1]. She showed that $\mathsf{C}(A)$ is a 2-Calabi-Yau triangulated category and has a cluster tilting object if $\mathsf{C}(A)$ is Hom-finite.

There exists a connection between $\underline{\mathsf{Sub}}\,\Pi(w)$ and cluster categories. In [2], for any element w in W and a reduced expression $\boldsymbol{w}$ of w, the authors constructed a finite dimensional algebra $A(\boldsymbol{w})$ and they showed that there exists a triangle equivalence $\mathsf{C}(A(\boldsymbol{w})) \simeq \underline{\mathsf{Sub}}\,\Pi(w)$.

In this talk, we first study a graded analogue of an existence of cluster tilting objects of $\underline{\mathsf{Sub}}\,\Pi(w)$. The orientation of Q gives a natural grading on the algebra $\Pi(w)$. We consider a triangulated category $\underline{\mathsf{Sub}}^{\mathbb{Z}}\Pi(w)$, which is a graded analogue of $\underline{\mathsf{Sub}}\,\Pi(w)$. We have the following theorem.

Theorem 1. *For any reduced expression $\boldsymbol{w}$ of w, the object $M(\boldsymbol{w}) \in \underline{\mathsf{Sub}}^{\mathbb{Z}}\Pi(w)$ is a silting object.*

In general, $M(\boldsymbol{w})$ is not a tilting object of $\underline{\mathsf{Sub}}^{\mathbb{Z}}\Pi(w)$. Under a certain condition on $\boldsymbol{w}$, $M(\boldsymbol{w})$ becomes a tilting object of $\underline{\mathsf{Sub}}^{\mathbb{Z}}\Pi(w)$. We call such conditions c-ending or c-starting.

Theorem 2. *Let $w \in W$ and $\boldsymbol{w}$ be a reduced expression of w. If $\boldsymbol{w}$ is c-ending or c-starting, then $M = M(\boldsymbol{w})$ is a tilting object of $\underline{\mathsf{Sub}}^{\mathbb{Z}}\Pi(w)$ and we have a triangle equivalence*

$$\mathsf{D}^{\mathrm{b}}(\mathsf{mod}\,\underline{\mathsf{End}}^{\mathbb{Z}}_{\Pi(w)}(M)) \simeq \underline{\mathsf{Sub}}^{\mathbb{Z}}\Pi(w).$$

Finally, we compare the equivalence obtained by $M(\boldsymbol{w})$ and the equivalence of Amiot-Reiten-Todorov.

Theorem 3. *Let $w \in W$ and $\boldsymbol{w}$ be a reduced expression of w. If $\boldsymbol{w}$ is c-ending, then $\underline{\mathsf{End}}^{\mathbb{Z}}_{\Pi(w)}(M(\boldsymbol{w})) = A(\boldsymbol{w})$ holds and we have the following commutative diagram up to isomorphism of functors*

$$\begin{tikzcd}
\mathsf{D}^{\mathrm{b}}(\mathsf{mod}A(\boldsymbol{w})) \arrow[r, "\simeq"] \arrow[d, "\pi"] & \underline{\mathsf{Sub}}^{\mathbb{Z}}\Pi(w) \arrow[d, "\mathrm{Forget}"] \\
\mathsf{C}(A(\boldsymbol{w})) \arrow[r, "\simeq"] & \underline{\mathsf{Sub}}\,\Pi(w),
\end{tikzcd}$$

where π is a canonical triangle functor.

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On mutation of τ-tilting modules

Yingying Zhang

Mutation of τ-tilting modules is a basic operation to construct a new support τ-tilting module from a given one by replacing a direct summand. The aim of this paper is to give a positive answer to the question posed in [AIR, Question 2.31] about mutation of τ-tilting modules.

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t-structures and silting objects

Takahide Adachi

In this talk, we study a connection between t-structures and silting objects. The notion of t-structures was introduced by Beilinson, Bernstein and Deligne (see [2]) and appears in many branches of mathematics. To understand t-structures from the viewpoint of representation theory of algebras, Keller-Vossieck introduced the notion of silting objects which is a generalization of the notion of tilting objects. They showed the following theorem. For a triangulated category $\mathcal{T}$, we denote by $\mathrm{silt}\,\mathcal{T}$ the set of isomorphism classes of basic silting objects of $\mathcal{T}$ and $\text{t-str}\,\mathcal{T}$ the set of all bounded t-structures on $\mathcal{T}$.

Theorem 1. [3] *Let Q be a Dynkin quiver and $\Lambda := KQ$ the path algebra over a field K. Then there is a bijection*

$$\mathrm{silt}\,\mathsf{K}^{\mathrm{b}}(\mathsf{proj}\,\Lambda) \longrightarrow t\text{-str}\,\mathsf{D}^{\mathrm{b}}(\mathsf{mod}\,\Lambda).$$

Our aim of this talk is to give a generalization of Theorem 1. Let $\mathcal{T}$ be a Hom-finite Krull-Schmidt triangulated category with the shift functor [1] and let $\mathcal{U}$ be a thick subcategory of $\mathcal{T}$ with a silting object M. Assume that $(\mathcal{T}_M^{\leq 0}, \mathcal{T}_M^{\geq 0})$ is a bounded t-structure on $\mathcal{T}$, where

$$\mathcal{T}_M^{\leq 0} := \{X \in \mathcal{T} \mid \mathrm{Hom}_{\mathcal{T}}(M, X[i]) = 0 \text{ for all integers } i > 0\},$$
$$\mathcal{T}_M^{\geq 0} := \{X \in \mathcal{T} \mid \mathrm{Hom}_{\mathcal{T}}(M, X[i]) = 0 \text{ for all integers } i < 0\}.$$

Then the correspondence $N \mapsto (\mathcal{T}_N^{\leq 0}, \mathcal{T}_N^{\geq 0})$ gives a well-defined map $\mathrm{silt}\,\mathcal{U} \to \text{t-str}\,\mathcal{T}$. A triangulated category $\mathcal{T}$ is said to be *silting-discrete* (see [1]) if, for each silting object M and positive integer l, the set $\{N \in \mathrm{silt}\,\mathcal{T} \mid M \geq N \geq M[l]\}$ is finite, where $M \geq N$ means $\mathrm{Hom}_{\mathcal{T}}(M, N[k]) = 0$ for all positive integers k. Note that, if Q is a Dynkin quiver, then $\mathsf{K}^{\mathrm{b}}(\mathsf{proj}\,KQ)$ is silting-discrete. Our main result is the following theorem.

Theorem 2. *If $\mathcal{U}$ is silting-discrete, then there is a bijection*

$$\mathrm{silt}\,\mathcal{U} \longrightarrow t\text{-str}\,\mathcal{T}$$

given by $N \mapsto (\mathcal{T}_N^{\leq 0}, \mathcal{T}_N^{\geq 0})$.

The following theorem plays an important role when we show Theorem 2.

Theorem 3. *The following are equivalent.*

(1) *$\mathcal{U}$ is silting-discrete.*

(2) *Each bounded t-structure $(\mathcal{C}^{\leq 0}, \mathcal{C}^{\geq 0})$ on $\mathcal{T}$ is given by a silting object N (i.e., $\mathcal{C}^{\leq 0} = \mathcal{T}_N^{\leq 0}$).*

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Ringel duality and Recollements

Hiroyuki Minamoto

It has been known by Cline-Parshall-Scott [2] that a quasi-hereditary algebra Λ is obtained by gluing its residue fields $\Gamma_1, \ldots, \Gamma_n$. More precisely, there are a sequence of recollements

$$
\begin{aligned}
&\mathcal{D}^{\mathrm{b}}(\Gamma_1) \equiv \mathcal{D}^{\mathrm{b}}(\Lambda_2) \equiv \mathcal{D}^{\mathrm{b}}(\Gamma_2),\\
&\mathcal{D}^{\mathrm{b}}(\Lambda_2) \equiv \mathcal{D}^{\mathrm{b}}(\Lambda_3) \equiv \mathcal{D}^{\mathrm{b}}(\Gamma_3),\\
&\mathcal{D}^{\mathrm{b}}(\Lambda_3) \equiv \mathcal{D}^{\mathrm{b}}(\Lambda_4) \equiv \mathcal{D}^{\mathrm{b}}(\Gamma_4),\\
&\qquad\vdots \qquad\qquad \vdots \qquad\qquad \vdots\\
&\mathcal{D}^{\mathrm{b}}(\Lambda_{n-1}) \equiv \mathcal{D}^{\mathrm{b}}(\Lambda) \equiv \mathcal{D}^{\mathrm{b}}(\Gamma_n).
\end{aligned} \tag{3}
$$

However, for a finite dimensional algebra Λ, existence of such a sequence of recollements does not ensure that it is quasi-hereditary. Recently, Krause [6] determined the condition for a sequence of recollements of residue fields which ensure that Λ is quasi-hereditary.

Let Λ be a quasi-hereditary algebra. Since its Ringel dual $\mathsf{R}(\Lambda)$ is a quasi-hereditary algebra with the reverse order on the idempotents $e_1, e_2, \ldots, e_n$, there is a sequence of recollements

$$
\begin{aligned}
&\mathcal{D}^{\mathrm{b}}(\Gamma_n) \equiv \mathcal{D}^{\mathrm{b}}(\Lambda'_2) \equiv \mathcal{D}^{\mathrm{b}}(\Gamma_{n-1}),\\
&\mathcal{D}^{\mathrm{b}}(\Lambda'_2) \equiv \mathcal{D}^{\mathrm{b}}(\Lambda'_3) \equiv \mathcal{D}^{\mathrm{b}}(\Gamma_{n-2}),\\
&\mathcal{D}^{\mathrm{b}}(\Lambda'_3) \equiv \mathcal{D}^{\mathrm{b}}(\Lambda'_4) \equiv \mathcal{D}^{\mathrm{b}}(\Gamma_{n-3}),\\
&\qquad\vdots \qquad\qquad \vdots \qquad\qquad \vdots\\
&\mathcal{D}^{\mathrm{b}}(\Lambda'_{n-1}) \equiv \mathcal{D}^{\mathrm{b}}(\mathsf{R}(\Lambda)) \equiv \mathcal{D}^{\mathrm{b}}(\Gamma_1).
\end{aligned}
$$

In this note, we show that we can get this sequence from the sequence (3) by categorical operation. In case of the number n of the idempotents is 2 (so the sequence consists of single recollement), this operation is nothing but the reflection due to P. Jorgensen [3]. This observation gives a look of the results of Krause [5] that twice of the Ringel duality is the Serre duality [1].

Our observation enable us to generalize a notion of Ringel duality for finite dimensional algebra equipped with an appropriate sequence of recollements by using the results of Koenig-Yang [4].

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[1] For a special class of quasi-hereditary algebras, this was conjectured by Kapranov and proved by Beilinson-Bezrukavnikov-Mirković [1].

Extension groups between atoms and classification of localizing subcategories

Ryo Kanda

For a commutative noetherian ring R, Gabriel [1] gave a classification of localizing subcategories of the category $\operatorname{Mod} R$ of R-modules, and showed a property of them:

Theorem 1 ([1, Proposition VI.2.4])**.** *Let R be a commutative noetherian ring. Then there is a bijection*

$$\{\, \textit{localizing subcategories of } \operatorname{Mod} R \,\} \xrightarrow{\sim} \{\, \textit{specialization-closed subsets of } \operatorname{Spec} R \,\}$$

given by $\mathcal{X} \mapsto \bigcup_{M \in \mathcal{X}} \operatorname{Supp} M$.

Theorem 2 ([1, Proposition V.5.10])**.** *Let R be a commutative noetherian ring. Then every localizing subcategory of $\operatorname{Mod} R$ is closed under injective envelopes.*

Theorem 1 has been generalized to locally noetherian Grothendieck category $\mathcal{A}$ in terms of the atom spectrum $\operatorname{ASpec} \mathcal{A}$:

Theorem 3 ([2, Theorem 3.8], [4, Corollary 4.3], and [3, Theorem 5.5])**.** *Let $\mathcal{A}$ be a locally noetherian Grothendieck category. Then there is a bijection*

$$\{\, \textit{localizing subcategories of } \mathcal{A} \,\} \xrightarrow{\sim} \{\, \textit{localizing subsets of } \operatorname{ASpec} \mathcal{A} \,\}$$

given by $\mathcal{X} \mapsto \bigcup_{M \in \mathcal{X}} \operatorname{ASupp} M$.

On the other hand, Theorem 2 does not necessarily hold for a locally noetherian Grothendieck category. Even in the case of the module category Λ of a noncommutative artinian ring Λ, or in the case of the category $\operatorname{GrMod} A$ of $\mathbb{Z}$-graded modules over a commutative noetherian positively graded ring A, we can construct a localizing subcategory which is not closed under injective envelopes.

In this talk, we determine which localizing subcategories are closed under injective envelopes, in terms of atom spectrum. We introduce the *extension groups between atoms*, denoted by $\operatorname{Ext}^i_{\mathcal{A}}(\alpha, \beta)$ for $\alpha, \beta \in \operatorname{ASpec} \mathcal{A}$, and obtain the following result.

Theorem 4. *Let $\mathcal{A}$ be a locally noetherian Grothendieck category. Then a localizing subcategory $\mathcal{X}$ of $\mathcal{A}$ is closed under injective envelopes if and only if the corresponding localizing subset $\Phi := \bigcup_{M \in \mathcal{X}} \operatorname{ASupp} M$ of $\operatorname{ASpec} \mathcal{A}$ has the following property: if $\alpha \in \operatorname{ASpec} \mathcal{A}$ and $\beta \in \Phi$ satisfy $\operatorname{Ext}^1_{\mathcal{A}}(\alpha, \beta) \neq 0$, then $\alpha \in \Phi$.*

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Classifying dense subcategories of exact categories via Grothendieck groups

Hiroki Matsui

Let $\mathcal{C}$ be a category. Then classifying subcategories of $\mathcal{C}$ means finding a bijection

$$\{\dots \text{ subcategories of } \mathcal{C}\}$$
$$\downarrow\uparrow$$
$$A,$$

where A is a set which is easier to understand.

The classification of subcategories is an important approach to understand the category $\mathcal{C}$ and has been studied in various areas of mathematics, for example, stable homotopy theory, commutative/noncommutative ring theory, algebraic geometry, and modular representation theory of finite groups.

Let $\mathcal{A}$ be an additive category and $\mathcal{X}$ a full additive subcategory of $\mathcal{A}$. We say that $\mathcal{X}$ is *additively closed* if it is closed under direct summands, and $\mathcal{X}$ is *dense* if any object in $\mathcal{A}$ is a direct summand of some object of $\mathcal{X}$. We can easily show that $\mathcal{X}$ is additively closed if and only if $\mathcal{X} = \mathsf{add}\mathcal{X}$ and $\mathcal{X}$ is dense if and only if $\mathcal{A} = \mathsf{add}\mathcal{X}$. Here, $\mathsf{add}\mathcal{X}$ denotes the smallest full additive subcategory closed under taking direct summands. Therefore, for any full additive subcategory $\mathcal{X}$ of $\mathcal{A}$, $\mathcal{X}$ is a dense subcategory of $\mathsf{add}\mathcal{X}$ and $\mathsf{add}\mathcal{X}$ is an additively closed subcategory of $\mathcal{A}$. Hense, to classify additive subcategories, it suffices to classify additively closed ones and dense ones.

Classification of additively closed subcategories has deeply been studied so far. For instance, Serre subcategories of module categories over commutative noetherian rings by Gabriel [1], thick subcategories of perfect complexes over commutative noetherian rings by Hopkins and Neeman [2, 3].

On the other hand, Thomason [4] classified dense triangulated subcategories of triangulated categories via their Grothendieck groups.

Theorem 1 (Thomason)**.** *Let $\mathcal{T}$ be an essentially small triangulated category. Then there is a bijection*

$$\{\textit{dense triangulated subcategories of } \mathcal{T}\}$$
$$\downarrow\uparrow$$
$$\{\textit{subgroups of } \mathrm{K}_0(\mathcal{T})\}.$$

In this talk, we discuss classifying certain class of dense subcategories of exact categories, which we call dense coherent subcategories, via their Grothendieck groups.

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Wall-crossing between stable and co-stable ADHM data

大川　領

本公演では射影平面 $\mathbb{P}^2$ 上のある種のベクトル束のモジュライから定義される Nekrasov 分配関数と呼ばれる母関数について紹介する．このモジュライが箙の道代数という多元環の表現のモジュライとして構成されること, 及びこの多元環の表現を用いた解析により母関数の関数等式が導かれることを説明する．

Nekrasov 分配関数 Z は物理学者 Nekrasov により導入され, 射影平面上の枠付き連接層のモジュライ空間上で同変コホモロジー類を積分することにより定義される．

$$Z = \sum_{n=0}^{\infty} q^n \int_{M(r,n)} \psi$$

ここで $M(r,n)$ は射影平面上の階数 r, 第 2 Chern 類が n の枠付き連接層 (E,Φ) のモジュライ空間, ψ は $M(r,n)$ 上の同変コホモロジー類で物理理論に応じて適切なものをとる．積分はモジュライ空間 $M(r,n)$ の持つ代数的トーラスの作用による局所化の方法によって定義される．

中島-吉岡は Nekrasov 分配関数を用いて代数曲面の Donaldson 不変量と Seiberg-Witten 不変量についての Witten の予想した関係式を示した．

一方, 物理学者 Ito-Maruyoshi-Okuda は, $p>1$ に対して A_{p-1} 型の ALE 空間と商スタック $[\mathbb{C}^2/\mathbb{Z}_p]$ を考察した．この二つの代数曲面は ALE 空間上のインスタントンモジュライの特異点解消であり, 物理的には両者ともインスタントンモジュライ上の積分を計算するべきものである．当然, 二つの分配関数の差は小さいことが予想され, その差を求めることは自然な問題として提起される．

この発表では $p=1$ の場合, つまり ALE 空間と商スタックがともに $\mathbb{C}^2$ である場合を扱う．枠付き連接層のモジュライ空間には ADHM データと呼ばれる行列の組による記述が知られており, この記述を用いると $p=1$ の場合でもインスタントンモジュライの特異点解消が二つ得られる．即ち安定な ADHM データのモジュライと余安定な ADHM データのモジュライである．両者は多様体としては同型であるが, 異なるトーラス作用を持つために異なる分配関数を定める．主結果として二つの分配関数の満たす関数等式を導出した．

証明は望月拓郎氏の開発した壁越え公式をもとにする中島-吉岡の方法を踏襲した．これまで考察されなかった摩天楼層が枠付き連接層の安定性を崩すような壁越え現象を調べることにより主結果を得た．同様の方法により $p=2$ の場合に, Ito-Maruyoshi-Okuda の提案した予想も導けることを確認した．

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Weighted projective lines and Riemann surfaces

Helmut Lenzing, Paderborn

In this talk we work over the base field of complex numbers. We start by reviewing the relationship between weighted projective lines and smooth projective curves, equivalently, compact Riemann surfaces. There are three cases to consider:

(1) *Euler characteristic* > 0. Here, each weighted projective line with three weights is isomorphic to a quotient of the ordinary projective line (= Riemann sphere) by a polyhedral group, i.e. a finite subgroup of PSL(2,C).

(2) *Euler characteristic* $= 0$. Here, each weighted projective line (then of tubular type) arises as the quotient of a smooth elliptic curve by a cyclic group of order 2, 3, 4, or 6. This uses unpublished work with Meltzer from 2004, alternatively the detailed account by Chen-Chen-Zhou (2015).

(3) *Euler characteristic* < 0. I will discuss the Bundgaard-Nielsen-Fox theorem (with additions by Chau and Mennicke) giving a positive answer to an old conjecture by Fenchel.

In modern language the theorm states: Each weighted projective line $\mathbb{X}$ (more generally, each weighted smooth projective curve) arises as a quotient M/G, where M is a compact Riemann surface and G is a finite subgroup G of $\mathrm{Aut}(M)$. In more algebraic terms this states that the category $\mathrm{coh}\,\mathbb{X}$ of coherent sheaves on $\mathbb{X}$ arises as the skew group category of $\mathrm{coh}(M)$ with respect to the group action of G. I will discuss the strategy of proof, and present a number of illustrative examples.

Algebras sharing tha same poset of support τ-tilting modules with tree quiver algberas

Takuma Aihara and Ryoichi Kase

Let $\Lambda = kQ/I$ and $\Gamma = kQ'/I'$ be two basic algebras over an algebraically closed field k, where Q, Q' are finite quivers and I, I' are admissible ideals of kQ, kQ' respectively. We denote by $\mathsf{stilt}\Lambda$ (resp. $\mathsf{s}\tau\text{-}\mathsf{tilt}\Lambda$) the set of (isomorphism classes of) basic support tilting modules (resp. support τ-tilting modules) of Λ. Then there are partial orders on $\mathsf{stilt}\Lambda$ and $\mathsf{s}\tau\text{-}\mathsf{tilt}\Lambda$ ([1],[3]). D. Happel and L. Unger considered poset isomorphisms between two posets of support tilting modules of path algebras and gave us the following fascinating result.

Theorem 1. [2] *Let $\Lambda = kQ$ and $\Gamma = kQ'$ be two finite dimensional path algebras. Assume that there is a poset isomorphism between $\mathsf{stilt}\Lambda$ and $\mathsf{stilt}\Gamma$. Then the decorated quiver of Q is isomorphic to that of Q'. In particular, if Q' is a tree quiver, then Λ is isomorphic to Γ.*

In the case that Λ is a path algebra, $\mathsf{s}\tau\text{-}\mathsf{tilt}\Lambda$ coincides with $\mathsf{stilt}\Lambda$. Therefore it is natural to consider τ-tilting version of Happel-Unger's result. In this talk, we give a full characterization of finite dimensional basic algebras whose support τ-tilting posets are isomorphic to that of tree quiver algebras.

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Derived equivalences and smash products

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Throughout this talk $\Bbbk$ is a commutative ring and G is a group. Denote by G-GrCat the 2-category of G-graded small $\Bbbk$-categories and (weak) degree-preserving functors defined in [3]. In the paper [1] (a final form in [2]) we investigated when the orbit categories of a pair of derived equivalent small $\Bbbk$-categories with G-actions are derived equivalent. Here we consider the converse. By a 2-categorical Cohen-Montgomery duality proved in [3], this problem is reduced to the following. Let A and B be in G-GrCat, and assume that A and B are derived equivalent. Then under which condition are the smash products $A\#G$ and $B\#G$ derived equivalent? Our solution is as follows.

Theorem. Let A and B be as above, and assume that they are derived equivalent. If there exists a tilting subcategory $\mathcal{P}$ for A consisting of G-gradable complexes, and if B is equivalent in the 2-category G-GrCat to $\mathcal{P}$ with a G-grading defined by the canonical G-covering $(Q, 1)\colon A\#G \to A$, then the smash products $A\#G$ and $B\#G$ are derived equivalent.

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Algebras of partial triangulations

Laurent Demonet

This is a report on [1].

We introduce a class of finite dimensional algebras coming from partial triangulations of marked surfaces. A partial triangulation is a subset of a triangulation.

This class contains Jacobian algebras of triangulations of marked surfaces [3] (see also [2]) and Brauer graph algebras [4]. We generalize properties which are known or partially known for Brauer graph algebras and Jacobian algebras of marked surfaces. In particular, these algebras are symmetric when the considered surface has no boundary, they are at most tame, and we give a combinatorial generalization of flips or Kauer moves on partial triangulations which induces (in most cases) derived equivalences between the corresponding algebras. Notice that we also give an explicit formula for the dimension of the algebra.

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Local duality principle and Grothendieck's vanishing theorem

Tsutomu Nakamura

This a joint work with Prof. Yuji Yoshino. Let R be a commutative noetherian ring. We denote by $\mathcal{D} = D(\mathrm{Mod}\ R)$ the derived category of unbounded chain complexes of R-modules. It is known by Neeman's result [3] that there is a canonical bijection between the set of subsets of Spec R and the set of localizing subcategories of $\mathcal{D}$. We denote by $\mathcal{L}_W$ the localizing subcategory corresponding to a subset W of Spec R by Neeman's result. By a classical argument of the localization theory of triangulated categories, it turns out that there exists a right adjoint functor γ_W to the inclusion functor $\mathcal{L}_W \hookrightarrow \mathcal{D}$ (see [2]). If W is a specialization-closed subset of Spec R, then γ_W is nothing but the ordinary local cohomology functor $\mathrm{R}\Gamma_W$.

In this talk, I will show the following result which is a general principle behind the local duality theorem.

Theorem 1 (LD Principle). *We assume that the Krull dimension of R is finite. Let W be a subset of* Spec R. *Then there exists a canonical isomorphism*

$$\gamma_W \mathrm{RHom}_R(X, Y) \cong \mathrm{RHom}_R(X, \gamma_W Y)$$

for $X \in \mathcal{D}^-_{fg}$ and $Y \in \mathcal{D}^+$.

This is a generalization of Foxby's result [1, Proposition 6.1]. By using LD Principle, we can obtain the following result.

Theorem 2. *We assume that R admits a dualizing complex. Let W be a subset of* Spec R *and M a finitely generated R-module. Then $H^i(\gamma_W M) = 0$ for $i > \dim M$.*

This is a generalization of Grothendieck's vanishing theorem of ordinary local cohomology.

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BRICKS AND 2-TERM SIMPLE-MINDED COLLECTIONS

Sota Asai

We consider a finite-dimensional algebra A over an algebraically closed field K and the category of finite-dimensional A-modules $\operatorname{mod} A$.

An A-module M in $\operatorname{mod} A$ is called a *brick* if the endomorphism algebra $\operatorname{End}_A M$ is isomorphic to K, and a set $\mathcal{S}$ of isomorphic classes of bricks is called a set of *pairwise orthogonal isomorphic classes of bricks* if it satisfies that $\operatorname{Hom}_A(S_1, S_2) = 0$ for $[S_1] \neq [S_2] \in \mathcal{S}$. Ringel showed that there is a bijection between the all sets of pairwise orthogonal isomorphic classes of bricks and the wide subcategories [5], that is, the abelian exact subcategories of $\operatorname{mod} A$ closed under extensions.

A wide subctegory $\mathcal{W} \subset \operatorname{mod} A$ is called *left finite* if the minimum torsion class $\mathcal{T}(\mathcal{W})$ containing $\mathcal{W}$ is functorially finite, and we also use this term for the corresponding set of pairwise orthogonal isomorphic classes of bricks. This condition is very useful, because there are bijections between the following sets.

(a) The set $\mathrm{s}\tau\text{-tilt}\, A$ of isomorphic classes of support τ-tilting A-modules.
(b) The set $\text{f-tors}\, A$ of functorially finite torsion classes in $\operatorname{mod} A$.
(c) The set $\mathrm{f_L}\text{-wide}\, A$ of left finite wide subcategories of $\operatorname{mod} A$.
(d) The set $\mathrm{f_L}\text{-pobrick}\, A$ of left finite sets of pairwise orthogonal isomorphic classes of bricks.

The bijections between (a) and (b) are given by Adachi–Iyama–Reiten [1], (b) and (c) are given by Marks–Šťovíček [4], and (c) and (d) are the restriction of the Ringel's bijections.

In this talk, I will introduce two topics on these concepts.

First, I will give a direct description of the bijection from (a) to (d) obtained as above.

Theorem 1. *The following map* $\mathrm{s}\tau\text{-tilt}\, A \to \mathrm{f_L}\text{-pobrick}\, A$ *is well-defined and bijective; an isomorphic class* $[M]$ *of a support* τ*-tilting module* M *is sent to the set of isomorphic classes of indecomposable direct summands of* $M/\operatorname{rad}_B M$*, where* $B = \operatorname{End}_A M$*.*

This is the "nonindecomposable" version of the result of Demonet–Iyama–Jasso to appear in a new version of [3], and I will give a proof of this theorem.

I will also talk about the question on wide subcategories given by Marks–Šťovíček whether the torsion class $\mathcal{T}(\mathcal{W})$ is also functorially finite for any functorially finite wide subcategory $\mathcal{W}$ of $\operatorname{mod} A$. I will give an example of algebras A which have the negative answer to this question.

Second, I will give the bijections to $\mathrm{f_L}\text{-pobrick}\, A$ from the set $2\text{-smc}\, A$ of *2-term simple-minded collections* in $D^{\mathrm{b}}(\operatorname{mod} A)$, that is, the sets $\mathcal{X}$ of isomorphic classes in $D^{\mathrm{b}}(\operatorname{mod} A)$ with (i) $\operatorname{End}_{D^{\mathrm{b}}(\operatorname{mod} A)} X \cong K$ for $[X] \in \mathcal{X}$, (ii) $\operatorname{Hom}_{D^{\mathrm{b}}(\operatorname{mod} A)}(X_1, X_2) = 0$ if $[X_1] \neq [X_2] \in \mathcal{X}$, (iii) $\operatorname{Hom}_{D^{\mathrm{b}}(\operatorname{mod} A)}(X_1, X_2[n]) = 0$ for $[X_1], [X_2] \in \mathcal{X}$ and $n < 0$, (iv) $\mathcal{X}$ generates $D^{\mathrm{b}}(\operatorname{mod} A)$ as triangulated categories, and (v) the ith cohomology $H^i(X)$ is zero for $i \neq -1, 0$ and $[X] \in \mathcal{X}$. The following theorem is my result.

Theorem 2. *The following map* $2\text{-smc}\, A \to \mathrm{f_L}\text{-pobrick}\, A$ *is well-defined and bijective;* $\mathcal{X} \in 2\text{-smc}\, A$ *is sent to* $\mathcal{X} \cap (\operatorname{mod} A) \in \mathrm{f_L}\text{-pobrick}\, A$*.*

This theorem says each element in $\mathrm{f_L}\text{-pobrick}\, A$ can be uniquely completed to a 2-term simple-minded collection. 2-term simple-minded collections are actively investigated by Brüstle–Yang [2], and I would like to talk about the relationship between their results and mine.

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