Derived categories of finite dimensional algebras

Conference honoring Hideto Asashiba on the occasion of his 60th birthday

September 11-12, 2015 in Shizuoka University, Science Building C, Room 309

September 11 (Friday)

10:00 - 10:50 Claus Michael Ringel (University of Bielefeld)
Hereditary triangulated categories

11:10 - 12:00 Yoshiyuki Kimura (Kobe University)
Remarks on quantum unipotent subgroups and the dual canonical basis

13:00 - 13:50 Jan Schröer (University of Bonn)
Gorenstein algebras associated with Cartan matrices

14:10 - 15:00 Kota Yamaura (Yamanashi University)
Iwanaga-Gorenstein algebras of perfect style

15:40 - 16:30 Michio Yoshiwaki (Osaka City University)
A construction of Iwanaga-Gorenstein algebras of finite Cohen-Macaulay type

16:50 - 17:40 Junichi Miyachi (Tokyo Gakugei University)
Polygon of recollements

19:00 - 21:00 Banquet

September 12 (Saturday)

10:00 - 10:50 Steffen Koenig (University of Stuttgart)
Silting and simple-minded objects, derived and stable module categories

11:10 - 12:00 Ryoichi Kase (Nara Women's University)
Remarks on lengths of maximal green sequences for quivers of type $\tilde{A}_{n,1}$

13:00 - 13:50 Takahide Adachi (Nagoya University)
A combinatorial description of two-term tilting complexes for Brauer graph algebras

14:10 - 15:00 Yuya Mizuno (Nagoya University)
Classifying tilting complexes over preprojective algebras of Dynkin type

15:40 - 16:30 Hirotaka Koga (Tokyo Denki University)
Derived equivalences and Gorenstein projective dimension

16:50 - 17:40 Yuji Yoshino (Okayama University)
A little more about Auslander-Reiten duality and conjecture for MCM

Organizers

Hiroshi Nagase (Tokyo Gakugei University)
Michio Yoshiwaki (Osaka City University)
Osamu Iyama (Nagoya University)
Izuru Mori (Shizuoka University)

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Claus Michael Ringel (University of Bielefeld)

Title: Hereditary triangulated categories

Abstract: A triangulated category is called hereditary provided it is equivalent to the bounded derived category of a hereditary abelian category using an equivalence which respects the translation functor. Such a category has a very lucid additive structure. An old preprint of mine (from 1998) was devoted to these categories, but pretended to recover the triangulated structure of such a category. Based on suggestions by Michel Van den Bergh, Xiao-Wu Chen has now shown that a hereditary triangulated which is algebraic is triangle equivalent to the bounded derived category of a hereditary abelian category. It is an open question whether any hereditary triangulated category is algebraic. The lecture will provide a survey on some of the properties of hereditary triangulated categories. It is based on a forthcoming joint paper with Xiao-Wu Chen.

Yoshiyuki Kimura (Kobe University)

Title: Remarks on quantum unipotent subgroups and the dual canonical basis

Abstract: Quantum unipotent subgroup is the quantum coordinate ring of the unipotent subgroup associated with a finite subset which is defined by a Weyl group element of a symmetrizable Kac-Moody Lie algebra. By the works of Geiss-Leclerc-Schroer and Goodearl-Yakimov, it is known that it has a quantum cluster algebra structure. In this talk, I explain about the quantum coordinate ring of the pro-unipotent subgroup associated with a cofinite subset given by a Weyl group element and its compatibility with the dual canonical basis and the multiplicity-free property between the dual canonical basis element in the quantum unipotent subgroups and the one in the opposite. This talk is based on arXiv:1506.07912.

Jan Schröer (University of Bonn)

Title: Gorenstein algebras associated with Cartan matrices

Abstract: We discuss several classes of 1-Iwanaga-Gorenstein algebras arising naturally at the interface of representation theory of finite-dimensional algebras, Lie theory and cluster algebra theory. This is joint work with Christof Geiss and Bernard Leclerc.

Kota Yamaura (Yamanashi University)

Title: Iwanaga-Gorenstein algebras of perfect style

Abstract: For a finite dimensional algebra Λ over a field K, one can construct a graded self-injective algebra as the trivial extension $A = \Lambda \oplus \mathrm{Hom}_K(\Lambda, K)$. This graded self-injective algebras is a powerful tool in the study of representation theory of algebras. One important fact is the following Happel's equivalence, which provides representation theoretical relationships between algebras of finite global dimension and self-injective algebras.

Theorem.[Happel] If Λ has finite global dimension, then there exists the triangle-equivalence

$$\underline{\mathrm{mod}}^{\mathbb{Z}} A \simeq \mathsf{D}^{\mathrm{b}}(\mathrm{mod}\Lambda).$$

where $\underline{\mathrm{mod}}^{\mathbb{Z}} A$ is the stable category of $\mathbb{Z}$-graded A-modules and $\mathsf{D}^{\mathrm{b}}(\mathrm{mod}\Lambda)$ is the bounded derived category of $\mathrm{mod}\Lambda$.

In my talk, we will discuss a generalization of the above result to the following direction.

First we will introduce a new class of graded Iwanaga-Gorenstein algebras, which we call *IG algebras of perfect style*. An important example of IG algebras of perfect style is trivial extensions of the form

$\Lambda \oplus M$ where Λ is an algebra and M is a cotilting (Λ, Λ)-bimodule. So this class of IG algebras can be regarded as a generalization of the class of the trivial extensions of the form $\Lambda \oplus \mathrm{Hom}_K(\Lambda, K)$.

Secondly for a graded IG algebra A of perfect style, we consider the category $\mathrm{CM}^{\mathbb{Z}}A$ of graded Cohen-Macaulay A-modules. Since A is an IG algebra, $\mathrm{CM}^{\mathbb{Z}}A$ is a Frobenius cateogry, and so its stable category $\underline{\mathrm{CM}}^{\mathbb{Z}}A$ has a structure of triangulated category. We will show that if the 0-th subring of A has finite global dimension, then $\underline{\mathrm{CM}}^{\mathbb{Z}}A$ is triangle-equivalent to some bounded derived category.

This talk is a joint work with Hiroyuki Minamoto.

Michio Yoshiwaki (Osaka City University)

Title: A construction of Iwanaga-Gorenstein algebras of finite Cohen-Macaulay type

Abstract: This talk is based on joint work with H. Minamoto and K. Yamaura.

An Iwanaga-Gorenstein algebra is said to be *of finite Cohen-Macaulay type* if there are only finitely many indecomposable Cohen-Macaulay modules up to isomorphisms. Our aim is to give a construction of IG algebras of finite CM type.

Our strategy is the same as the case of self-injective algebras. Let Λ be a tilted algebra of Dynkin type, $\hat{\Lambda}$ the repetitive algebra of Λ and G an admissible group of automorphisms of $\hat{\Lambda}$. Then the orbit algebra $\hat{\Lambda}/G$ is a self-injective algebra of finite representation type (see [Riedtmann], [Tachikawa] and [Waschbüsch]). In this talk, we will construct IG algebras of finite CM type by using IG algebras of 1-perfect style introduced by Minamoto-Yamaura instead of the repetitive algebra $\hat{\Lambda}$. This construction requires Gabriel's covering techniques (for CM version), similar to the case of self-injective algebras.

Junichi Miyachi (Tokyo Gakugei University)

Title: Polygon of recollements

Abstract: We study a structure of subcategories which are called a polygon of recollements in a triangulated category. We describe categorical properties of polygon of recollments in triangulated categories, and study what kind of triangulated category has this structure.

Steffen Koenig (University of Stuttgart)

Title: Silting and simple-minded objects, derived and stable module categories

Abstract: Module categories can be seen as being generated by either the projective or the simple modules. The objects in the title are meant to allow for similar (or more general) statements for derived module categories. In the first part of the talk, I will explain these concepts and how they are related to each other and to t-structures and to co-t-structures. This part is based on joint work with Dong Yang. In the second part, based on joint work with Aaron Chan and Yuming Liu, I will discuss the situation for stable module categories, where much less is known.

Ryoichi Kase (Nara Women's University)

Title: Remarks on lengths of maximal green sequences for quivers of type $\tilde{A}_{n,1}$

Abstract: A maximal green sequence is a certain sequence of quiver mutations. T. Brüstle, G. Dupont and M. Pérotin showed that for an acyclic quiver, maximal green sequences are realized as maximal paths in the Hasse quiver of the poset of support tilting modules and conjectured that possible lengths of maximal green sequences form an interval in $\mathbb{Z}$. In this talk, we will consider possible lengths of maximal green sequences for quivers of type A or of type $\tilde{A}_{n,1}$.

Takahide Adachi (Nagoya University)

Title: A combinatorial description of two-term tilting complexes for Brauer graph algebras

Abstract: This talk is based on joint work with Takuma Aihara and Aaron Chan.

In Morita theory for derived categories, tilting complexes play an important role since they induce derived equivalences, which preserve many homological properties. Thus it is important to classify tilting complexes for a given algebra. In this talk, we give a classification of special tilting complexes, called two-term tilting complexes, for Brauer graph algebras.

Yuya Mizuno (Nagoya University)

Title: Classifying tilting complexes over preprojective algebras of Dynkin type

Abstract: In the representation theory of algebras, derived equivalences of algebras have been one of the central themes and extensively investigated. By Rickard's Morita theorem for derived categories, it is known that derived equivalences of rings are controlled by tilting complexes. In this talk, we discuss tilting complexes over preprojective algebras of Dynkin type. In particular, we explain a close relationship between tilting complexes and braid groups, and classify all tilting complexes. This is part of a joint work T.Aihara.

Hirotaka Koga (Tokyo Denki University) and Hoshino Mitsuo (University of Tsukuba)

Title: Derived equivalences and Gorenstein projective dimension

Abstract: Let $\mathcal{A}, \mathcal{B}$ be abelian categories with enough projectives. We denote by $\mathcal{P}_{\mathcal{A}}$ the full subcategory of $\mathcal{A}$ consisting of all projective objects in $\mathcal{A}$ and by $\mathcal{D}^{\mathrm{b}}(\mathcal{A})$ the derived category of complexes over $\mathcal{A}$ with bounded cohomology. In this note, we introduce the notion of complexes of finite Gorenstein projective dimension and show that a derived equivalence $F : \mathcal{D}^{\mathrm{b}}(\mathcal{A}) \xrightarrow{\sim} \mathcal{D}^{\mathrm{b}}(\mathcal{B})$ induces an equivalence between the full triangulated subcategories consisting of complexes of finite Gorenstein projective dimension provided that F satisfies the following condition: there exists $a > 0$ such that $\mathrm{Hom}_{D^{\mathrm{b}}(\mathcal{B})}(FP, Q[i]) = 0 = \mathrm{Hom}_{D^{\mathrm{b}}(\mathcal{B})}(Q, FP[i])$ for all $P \in \mathcal{P}_{\mathcal{A}}$ and $Q \in \mathcal{P}_{\mathcal{B}}$ unless $-a \leq i \leq a$.

Yuji Yoshino (Okayama University)

Title: A little more about Auslander-Reiten duality and conjecture for MCM

Abstract: TBA
