The Ninth China-Japan-Korea International Conference on Ring and Module Theory

Incheon, Republic of Korea · 14–19 August 2023

CHINA-JAPAN-KOREA INTERNATIONAL CONFERENCE ON RING AND MODULE THEORY

PROGRAMME and ABSTRACTS

CONFERENCE BOOKLET

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INU 인천대학교 INCHEON NATIONAL UNIVERSITY

Welcome

It is our great pleasure to welcome you to

The 9th China-Japan-Korea International Conference on Ring & Module Theory.

The conference will take place from August 14th to 19th, 2023 at

Incheon National University (INU)
Academy-ro 119, 22012 Incheon
Republic of Korea

This booklet has been put together to help you find your way through the conference and make the most of your stay in Incheon. Additional information can be found online at the URL:

http://cjk-korea.imweb.me/Home or http://cjk-korea.hanbat.ac.kr/Home

We thank the organizing committee from China and Japan for their help with the role of the organization in each countries:

CHINA - Nanqing DING · Jianlong CHEN · Fang LI · Quanshui WU

JAPAN - Shigeto KAWATA · Akira UEDA · Kota YAMAURA · Tomohiro ITAGAKI

During the week, the student volunteers can be recognized by the colored background of our badge. Please come and talk to them, or the Korean organizers, if you have any questions.

The Korean organizers

Nam Kyun KIM · Gyu Whan CHANG · Gangyong LEE · Il-Seung JANG

Sang Min CHUN · Jongwook BAECK · Hwankoo KIM · Tai Keun KWAK

Jung Wook LIM · Dong Yeol OH · Jun Seok OH · Mi Hee PARK

General information

Every lecture room will be equipped with a digital projector, a computer, and blackboards, and one of the student volumnteers will be there on hand to help. Speakers can bring their own laptop.

For general inquiries or technical problems (with projectors, Internet, etc.), please see the registration desk or contact one of the organizers or the student volunteers.

Internet access

Internet services are available for free. To access the Internet, turn on the Wireless LAN on your electronic devise in the area of the conference premises, and either use your eduroam account, if you have one, or select the network called

"Public Wifi Free"

This network is not secured, so you do not need a password to access it, though depending on your OS you might receive a general security warning for accessing an unsecured network.

Social Events

On Tuesday, August 15, 2023 at 18:30, there will be the conference banquet. Further details on the conference banquet will be announced.

Emergency numbers

In case of an emergency, you may contact one of the following:

- Police: 112
- Ambulance & Fire brigade: 119

For minor emergencies, you may also contact one of the following organizers:

- Nam Kyun Kim: +82 10 8637 7550
- Gyu Whan Chang: +82 10 2779 4801
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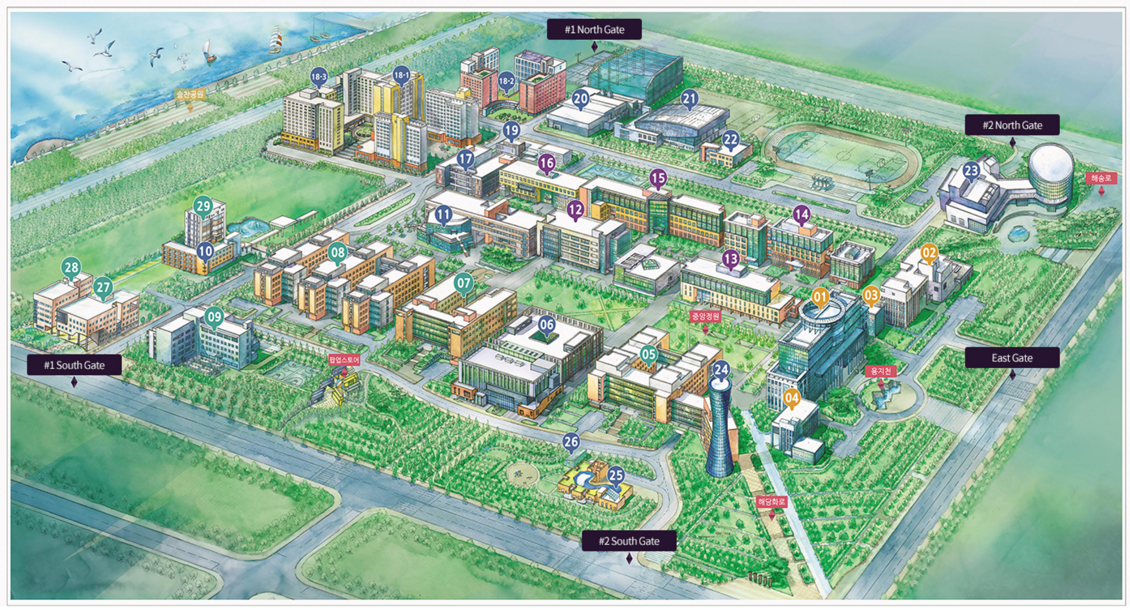
Where to have lunch or dinner



01 Main building	06 haksan Library	11 Welfare & Service Center (Cafeteria)	16 College of Arts and Physical Education	19 R&DB Foundation	24 Observatory	29 Division of Bioengineering
02 Faculty office building	07 College of Information Technology	12 Covention Center	17 Student Center	20 Sportss Center & GolfPractice Center	25 Chidren's Center	
03 PR Hall	08 College of Engineering	13 College of Business / School of Northeast Asian Studies	18-1 Domitory #1	21 Gymnasium	26 Greenhouse	
04 Computer & Information center (BM contents)	09 Center Laboratory	14 College of Business / School of Northeast Asian Studies	18-2 Domitory #2	22 Reserve officer traning center	27 2nd jointlaboratiry	
05 College of Natural Sciences/ College of Life Sciences and Bioengineering	10 Guest House	15 College of Humanities	18-3 Domitory #3	23 Auditorium & Performance Hall	28 College of Urban Science	

- Student Cafeteria	11	lunch : 10:30-14:00 dinner : 17:00-18:30
- Faculty & Staff Cafetaria	02	lunch : 11:30-13:30 dinner : 17:00-18:30
- 샹차이 (Chinese restaurant)	02	
- Kong's Rice Ball	17	
- Mom's Touch (Burger)	18-3	
- Convenience Stores	11, 13, 15	
- Cafe Dream	11	08:30-17:30

There are some restaurants outside of the university campus along the pathway behind the building no. 06. Moreover, we list the following two huge "Complex Malls" where you can find not only various restaurants and cafes but also pharmacies and drug stores, located near the conference venue.

- Times Space, Harmony-ro 158
- Triple Street, Songdogwahak-ro 16beon-gil 33

Invited Speakers

- Hongxing Chen (Capital Normal University, China)
- Alfred Geroldinger (University of Graz, Austria),
- Jiangsheng Hu (Jiangsu University of Technology, China),
- Byung Gyun Kang (POSTECH, Republic of Korea),
- Sijong Kwak (KAIST, Republic of Korea),
- Tsit Yuen Lam (University of California, Berkeley, USA),
- Hiroki Matsui (Tokushima University, Japan),
- Yuya Mizuno (Osaka Metropolitan University, Japan),

Plenary Speakers

- Min Huang (Sun Yat-sen University, China)
- Ayako Itaba (Tokyo University of Science, Japan)
- Ryo Kanda (Osaka Metropolitan University, Japan)
- Li Liang (Lanzhou Jiaotong University, China)
- Hwankoo Kim (Hoseo University, Republic of Korea)
- Jae-Hoon Kwon (Seoul National University, Republic of Korea)
- Pace P. Nielsen (Brigham Young University, USA)
- Michal Ziembowski (Warsaw University of Technology, Poland)

List of participants

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Jongwook Baeck (Hanbat National University)
Hyungtae Baek (Kyungpook National University)
Samruam Baupradist (Chulalongkorn University)
Sylvain Carpentier (Seoul National University)
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Jingheng Zhou (Shanghai University of Finance and Economics)
Mengmeng Zhou (Nanjing Xiaozhuang University)
Yukun Zhou (Southeast University)
Shenglin Zhu (Fudan University)
Michal Ziembowski (Warsaw University of Technology)

Monday (August 14)

	- Session **A** -	- Session **B** -	- Session **C** -
Chair	**Nam Kyun Kim**	**Mi Hee Park**	
9:00		Opening	
9:30-10:30		Byung Gyun Kang *Analytic closure (p.04)*	
10:30-11:10		Pace P. Nielsen *Chains of idempotents and bounded generation of SL2 (p.16)*	
11:10-11:30		Coffee Break	
11:30-11:55	Andreas Reinhart *Factorization into valuation ideals and valuation elements (p.66)*	Fitriani *The Centralizing Maps on Modules by using Idealization of Modules (p.27)*	———
11:55-12:20	Dong Yeol Oh *Factorization in the (composite) Hurwitz rings (p.56)*	Lixin Mao *Homological properties of modules over trivial ring extensions (p.51)*	———
12:20-14:30		Lunch Break	
Chair	**Daniel Smertnig**	**Cosmin S. Roman**	
14:30-14:55	Kazuho Ozeki *The first Hilbert coefficient and the reduction number of stretched ideals (p.58)*	Gangyong Lee *On polyform modules (p.48)*	———
14:55-15:20	Mara Pompili *On class group of upper cluster algebras (p.63)*	Qingyu Shao *On Gorenstein global and Gorenstein weak global dimensions (p.70)*	———
15:20-15:45	Balint Rago *On the arithmetic of orders in algebraic number fields (p.65)*	Kaito Kimura *On the Auslander-Reiten conjecture for normal rings (p.41)*	———
15:45-16:30		Coffee Break	
16:30-16:55	Shinnosuke Ishiro *Some ring-theoretic properties of a local log-regular ring (p.37)*	Gahng Sahn Lee *Dirac reductions and classical W-algebras (p.47)*	———
16:55-17:20	Hyungtae Baek *$R[X]_A$ of zero-dimensional reduced rings (p.19)*	Satoshi Usui *Periodic dimensions for modules and algebras (p.75)*	———
17:20-17:45	Sylvain Carpentier *A new method to quantify Integrable Systems (p.21)*	Bokhee Im *Combinatorial superalgebras and their superproducts (p.36)*	———

Tuesday (August 15)

	- Session **A** -	- Session **B** -	- Session **C** -
Chair	**Gangyong Lee**	**Nanqing Ding**	**Yuya Mizuno**
9:30-10:30		Hongxing Chen *Derived recollement approach to Tachikawa's second conjecture (p.01)*	
10:30-11:10		Jae-Hoon Kwon *Crystal base of the negative half of the quantum superalgebra for* $\mathfrak{gl}_{m\|n}$ *(p.15)*	
11:10-11:30		Coffee Break	
11:30-11:55	Giulio Peruginelli *Polynomial Dedekind Domains and Stacked Pseudo-convergent Sequences (p.62)*	Daniel Smertnig *On noncommutative bounded factorization domains and prime rings (p.72)*	Promod Sharma *Inductive algebras for compact groups (p.71)*
11:55-12:20	Takanori Nagamine *Retracts of Laurent polynomial rings (p.54)*	Cosmin S. Roman *On the rudimentary property of rings (p.67)*	Seung-Il Choi *Modules of 0-Hecke algebras from posets (p.24)*
12:20-14:30		Lunch Break	
Chair	**Jung Wook Lim**	**Pace P. Nielsen**	**Jae-Hoon Kwon**
14:30-15:30		Tsit Yuen Lam *Stable Range One Theory for Rings and Elements (p.06)*	
15:30-16:10		Li Liang *A construction of model structure (p.13)*	
16:10-16:30		Coffee Break	
16:30-16:55	De Chuan Zhou *Some characterizations of w-Noetherian rings and SM rings (p.84)*	Fei Peng *On the set of all generalized Drazin invertible elements in a ring (p.61)*	Shunya Saito *Classifying Serre subcategories of the category of Maximal Cohen-Macaulay modules (p.68)*
16:55-17:20	Manoj Kumar Patel *Some aspect on various generalizations of h–Lifting Modules (p.60)*	Shuxian Xu *The minimal weak Drazin inverses (p.78)*	Izuru Mori *Classification of locally free sheaf bimodules of rank 2 over the projective line (p.53)*
17:20-17:45	Samruam Baupradist *On the modules having proper S-essential submodules (p.20)*	Yukun Zhou *Drazin inverses and pseudo core inverses of a sum of morphisms (p.87)*	Shigeto Kawata *Tensor products and almost split sequences for group rings (p.40)*
18:30		Banquet	

Wednesday (August 16)

	- Session **A** -	- Session **B** -	- Session **C** -
Chair		**Shigeto Kawata**	**Hiroki Matsui**
9:30-10:30		Hiroki Matsui *Spectra of triangulated categories and their applications (p.07)*	
10:30-11:10		Ryo Kanda *Elliptic algebras (p.12)*	
11:10-11:30		Coffee Break	
11:30-11:55	———	Chang Ik Lee *The structure of rings which are symmetric on the set of all idempotents (p.46)*	Yuta Kozakai *On support τ-tilting modules over group algebras (p.42)*
11:55-12:20	———	Hongying Chen *Rings and radicals related to n-primariness (p.22)*	Kyoichi Suzuki *Morita equivalences and relative stable equivalences of Morita type for the principal blocks of finite groups (p.73)*
12:20-14:30		Photo / Lunch Break	
14:30		Excursion	

Thursday (August 17)

	- Session **A** -	- Session **B** -	- Session **C** -
Chair	**Sijong Kwak**	**Jianlong Chen**	**Hongxing Chen**
9:30-10:30		Yuya Mizuno *Fans, simplicial complexes and polytopes in tilting theory (p.08)*	
10:30-11:10		Ayako Itaba Quantum projective planes finite over their centers and Beilinson algebras (p.10)	
11:10-11:30		Coffee Break	
11:30-11:55	Yuehui Zhang *Birkhoff-Pierce Problem and Riemann Hypothesis (p.83)*	Yuefeng Gao *The absorption laws and the reverse order laws of two kinds of generalized core inverses (p.31)*	Jingheng Zhou *Frobenius-Perron Theory of Finite Representations of Quivers (p.86)*
11:55-12:20	Ryo Ishizuka *On the relation between perfectoidisation and p-root closure (p.38)*	Youssef Zahir *On S-1-absorbing prime and weakly S-1-absorbing prime ideals (p.81)*	Tomohiro Itagaki *Hochschild homology dimension and a class of Hochschild extension algebras of truncated quiver algebras (p.39)*
12:20-14:30		Lunch Break	
Chair	**Michal Ziembowski**	**Byung Gyun Kang**	**Ayako Itaba**
14:30-15:30		Alfred Geroldinger *Rings, Modules, and the Transfer Krull Property (p.02)*	
15:30-16:10		Min Huang *Positivity for quantum cluster algebras from surfaces (p.09)*	
16:10-16:30		Coffee Break	
16:30-16:55	Laura Cossu *On the finiteness of certain factorization invariants (p.26)*	Liyun Wu *A new class of generalized inverses in rings with involution (p.77)*	Ryo Takahashi *Classifying t-structures of derived categories (p.74)*
16:55-17:20	Kota Yamaura *Tilting theory for finite dimensional 1-Iwanaga-Gorenstein algebras (p.79)*	Tai Keun Kwak *Structure of right APIP rings (p.44)*	Arashi Sakai *IE-closed subcategories (p.69)*
17:20-17:45	Xianhui Fu *Powers of ghost ideals (p.29)*	Jung Miao Kuo *Hirata separability, partial Galois extension and inner partial group action (p.43)*	Yasuaki Ogawa *K_0 of weak Waldhausen extriangulated categories (p.55)*

Friday (August 18)

	- Session **A** -	- Session **B** -	- Session **C** -
CHAIR		**Jiangsheng Hu**	**Akira Ueda**
9:30-10:30		Sijong Kwak *The Eisenbud-Goto regularity conjecture for homogeneous prime ideals (p.05)*	
10:30-11:10		Michal Ziembowski *On subalgebras of matrix algebra satisfying some polynomial identity (p.17)*	
11:10-11:30		COFFEE BREAK	
11:30-11:55	————	Masaki Matsuno *Classification of twisted algebras of 3-dimensional Sklyanin algebras (p.52)*	Shengyong Pan *Characterizations of standard derived equivalences of diagrams of dg categories and their gluings (p.59)*
11:55-12:20	————	Anna Cichocka *On some classes of subalgebras of Leavitt path algebras (p.25)*	Sin-Myung Lee *Super duality for quantum affine algebras of type A (p.49)*
12:20-14:30		LUNCH BREAK	
CHAIR		**Hwankoo Kim**	**Ryo Kanda**
14:30-14:55	————	Huai Wen Guo *Morita contexts for Hopf coquasi-module algebra (p.33)*	Yuya Otake *The n-torsionfreeness of syzygies of the residue field (p.57)*
14:55-15:20	————	Botong Gai *A Duality Theorem for Monoidal Hom Hopf Module Algebras (p.30)*	Norihiro Hanihara *Quotient singularities, higher hereditary algebras, and non-commutative regular algebras (p.35)*
15:20-15:45	————	Taolue Long *Recollements and n-cluster tilting subcategories (p.50)*	Yasuaki Gyoda *Torsion lattice of GLS path algebra and Cambrian lattice (p.34)*
15:45-16:30		COFFEE BREAK	
16:30-16:55	————	Xiaolei Zhang *On the uniformity in rings, modules and homological algebras (p.82)*	Dian Ariesta Yuwaningsih *Some Properties of Left r-Clean Bimodules (p.80)*
16:55-17:20	————	Arindam Ghosh *(m,n)-Jordan $*$-Centralizers and Their Generalizations over $*$-Prime and $*$-Semi-prime Rings (p.32)*	Nikken Prima Puspita *From Derivation on Module Theory to Co-derivation on Comodule Theory (p.64)*
17:20-17:45	————	————	Indah Emilia Wijayanti *On Differential Colocalizations on Coalgebras (p.76)*

Saturday (August 19)

	- Session **A** -	- Session **B** -	- Session **C** -
Chair		**Il-Seung Jang**	
9:30-10:30		Jiangsheng Hu *Cotorsion pairs and homological conjectures (p.03)*	
10:30-11:10		Hwankoo Kim *Ring-theoretic properties in a certain pullback (p.14)*	
11:10		Closing	

Contents

Invited talks

Derived recollement approach to Tachikawa's second conjecture

Hongxing Chen

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Tachikawa's second conjecture predicts that a finitely generated, orthogonal module over a finite-dimensional self-injective algebra is projective. This conjecture is an important part of the Nakayama conjecture. In this talk, I will report a series of advances on Tachikawa's second conjecture by addressing orthogonal generators over self-injective algebras from the view point of recollements of triangulated categories. In particular, Gorenstein-Morita algebras and (reduced) mirror-reflective algebras are introduced and equivalent characterizations of Tachikawa's second conjecture in terms of relative Gorenstein categories or stratifying ideals of algebras are presented. The talk is based on joint work on orthogonal modules with Changchang Xi and also with Ming Fang.

2020 Mathematics Subject Classification: 16E35, 18G80, 18G65

Keywords: Recollement, Triangulated category, Tachikawa's second conjecture

Rings, Modules, and the Transfer Krull Property

Alfred Geroldinger

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Krull monoids can be studied with ideal theoretic as well as with divisor theoretic tools. Indeed, a commutative and cancellative monoid H is Krull if one of the following equivalent conditions hold.

(a) H has a divisor theory.

(b) There is a divisor homomorphism from H to a free abelian monoid.

(c) H is completely integrally closed and satisfies the ascending chain condition on divisorial ideals.

A commutative integral domain is a Krull domain if its monoid of nonzero elements is a Krull monoid. The concept of Krull monoids and domains goes back to the first half of the previous century. It was then successfully generalized to non-commutative rings as well as to commutative rings with zero-divisors.

A monoid homomorphism $\theta\colon H \to B$ is called a transfer homomorphism if it is surjective up to units and allows to lift factorizations. Arithmetic invariants (which measure the deviation of a monoid from being factorial) can be studied in the (oftentimes simpler) monoid B and then they can be pulled back by the homomorphism θ to the monoid H.

It can easily be seen that a Krull monoid allows a transfer homomorphism to a monoid of zero-sum sequences over its class group, whose arithmetic can be studied with methods from additive combinatorics. In the last decade it turned out that various classes of monoids and domains (which need neither be commutative nor cancellative nor completely integrally closed) allow a transfer homomorphism to a commutative Krull monoid and hence to a monoid of zero-sum sequences. Such monoids are called transfer Krull.

In this survey talk we provide examples of Krull monoids and of transfer Krull monoids, that are not Krull, which arise in ring and module theory, and we discuss some of their main algebraic and arithmetic properties.

2020 Mathematics Subject Classification: 13A15, 13F05, 20M12, 20M13, 11B30

Keywords: Krull monoids, Krull domains, class groups, sets of lengths

Cotorsion pairs and homological conjectures

Jiangsheng Hu

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The notion of cotorsion pairs goes back to the 1970s when it was introduced by Salce in the case of abelian groups. Cotorsion pairs have been used to study covers and envelopes, particularly in the proof of the flat cover conjecture. They have also been used in homotopy theory and in tilting theory. In this talk, we present some homological properties of cotorsion pairs with a view to applications to the finitistic dimension conjecture, the Wakamatsu tilting conjecture, and the Auslander and Reiten conjecture. This talk is based on several joint works with L.W. Christensen, N.Q. Ding, S. Estrada, H.H. Li, Y.X. Li, P. Thompson and J. Wang.

References

[1] L.W. Christensen, N.Q. Ding, S. Estrada, J.S. Hu, H.H. Li and P. Thompson, The singularity category of an exact category applied to characterize Gorenstein schemes, Quarterly J. Math., 74(1) (2022) 1-27.

[2] J. Wang, Y.X. Li and J.S. Hu, When the kernel of a complete hereditary cotorsion pair is the additive closure of a tilting module, J. Algebra 530 (2019) 94-113.

[3] J. Wang, Y.X. Li, J.Y. Wu and J.S. Hu, Auslander conditions and tilting-like cotorsion pairs, to appear in J. Algebra.

2020 Mathematics Subject Classification: 16E65, 16E10, 18G25, 16G10

Keywords: cotorsion pair, tilting module, Gorenstein projective module, singularity category, Auslander condition, finitistic dimension

Analytic closure

Byung Gyun Kang

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Let V be a valuation domain with the value group the reals and K be its quotient field. Let W be a one-dimensional valuation extension of V to the algebraic closure of K. We show that every power series over V, which is not an associate of a constant, is an infinite product of countably many primitive power series over W with the initial degree 1. Let $\widehat{W}$ be the completion of W. Also we show that every power series over $\widehat{W}$, which is not an associate of a constant, has a zero in $\widehat{W}$ and is an infinite product of countably many primitive power series of the form xa, $a \in \widehat{W}$. This is a joint work with M.H. Park

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2020 Mathematics Subject Classification: 13A05, 13F25, 13F30

Keywords: Valuation domain, power series ring, infinite product

The Eisenbud-Goto regularity conjecture for homogeneous prime ideals

Sijong Kwak

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The Eisenbud-Goto regularity conjecture for homogeneous prime ideals in polynomial rings is one of the long-standing conjectures in Algebraic Geometry and Commutative Algebra. There is one to one correspondence between homogeneous prime ideals and projective irreducible varieties. Various methods have been developed according to categories of objects.

It is well known that the Eisenbud-Goto regularity conjecture is true for projective curves, smooth surfaces, smooth threefolds in $\mathbb{P}^5$, and toric varieties of codimension two. Since Jason McCullough and Irena Peeva constructed counterexamples in 2018 for the first time, there is a mysterious dichotomy between smooth varieties and singular varieties. In this talk, we'd like to present a survey on recent developments on regularity of homogeneous prime ideals in polynomial rings.

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2020 Mathematics Subject Classification: 14N05, 13D02

Keywords: homogeneous prime ideal, projective variety, minimal free resolution, Castelnuovo-Mumford regularity, Eisenbud-Goto conjecture

Stable Range One Theory for Rings and Elements

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The theory of stable range for rings has a long history, originating from the work of H. Bass in 1964 on the stability properties of the general linear group $\mathrm{GL}_n(S)$ over a ring S. While this work marked the beginning of the subject of algebraic K-theory, it has incidentally exerted a large influence on the theory of commutative and non-commutative rings. The case of rings of stable range one ("sr1 rings") is of special interest, due to its various connections with the problems of cancellation, substitution, and unique generation, etc. In this talk, the speaker will give a quick report on a relatively new (c. 2004) theory of "elements of stable range one" in arbitrary rings, and tell the story of how this work has led him to the discovery of a new determinantal identity for three matrices over any commutative ring. This is recent joint work with Dinesh Khurana.

2020 Mathematics Subject Classification:

Keywords: Noncommutative rings, stable range theory, determinantal identities

Spectra of triangulated categories and their applications

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For a tensor triangulated category $(\mathcal{T}, \otimes, \mathbf{1})$, Balmer ([1]) introduced the ringed space $\mathrm{Spec}_{\otimes}(\mathcal{T})$, which is called the tensor triangular spectrum of $\mathcal{T}$. As tensor triangulated categories are ubiquitous in mathematics, this gives a unified approach to studying commutative algebra, algebraic geometry, representation theory, and so on via algebro-geometric methods. This theory is called tensor triangular geometry. For example, Balmer ([1]) proved the following reconstruction theorems in algebraic geometry and representation theory:

- Let X be a noetherian scheme. Then the tensor triangular spectrum of the perfect derived category $\mathrm{D}^{\mathrm{perf}}(X)$ of X is isomorphic to X: $\mathrm{Spec}_{\otimes}(\mathrm{D}^{\mathrm{perf}}(X)) \cong X$.
- Let G be a finite group and k a field. Then the tensor triangular spectrum of the stable category $\mathrm{stmod}(kG)$ of kG is isomorphic to $\mathrm{Proj}\,\mathrm{H}^{\bullet}(G;k)$: $\mathrm{Spec}_{\otimes}(\mathrm{stmod}(kG)) \cong \mathrm{Proj}\,\mathrm{H}^{\bullet}(G;k)$.

Tensor triangular geometry has been successful and well-studied so far in many areas, whereas it cannot be directly applied to triangulated categories without tensor structures. Therefore it is a natural and important problem to develop an analogous theory of tensor triangular geometry for triangulated categories without using tensor structures. In one of these attempts, I have recently introduced the ringed space $\mathrm{Spec}_{\triangle}(\mathcal{T})$, which is called the triangular spectrum of $\mathcal{T}$ for a triangulated category $\mathcal{T}$ ([2,3]).

In this talk, we develop the theory of triangular spectra and apply it to representation theory, algebraic geometry, and commutative algebra. The results include several known or unknown reconstruction theorems.

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2020 Mathematics Subject Classification: 13C60, 14A15, 18G80, 20C05

Keywords: derived category, stable category, tensor triangular spectrum, triangular spectrum, triangulated category

Fans, simplicial complexes and polytopes in tilting theory

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The notion of tilting objects is basic to study the structure of a given derived category. The class of silting objects gives a completion of the class of tilting objects from the point of view of mutation, and they correspond bijectively with other important objects in the derived category. The subset of 2-term silting complexes enjoys especially nice properties, which is closely related to τ-tilting theory and cluster theory. In this talk, we discuss the notion of g-simplicial complexes, g-polytopes and g-fans, which is defined from 2-term silting complexes. We study several properties of these three objects. In particular, we give tilting theoretic interpretations of the h-vectors and Dehn-Sommerville equations of the g-simplicial complex. Moreover, we discuss the convexity of the g-polytope and its dual polytope. We also discuss a classification of rank 2 g-fans. This is joint work with Aoki-Higashitani-Iyama-Kase.

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2020 Mathematics Subject Classification: 16G10

Keywords: silting complexes, polytopes, fans

Plenary talks

Positivity for quantum cluster algebras from surfaces

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We provide a combinatorial formula for quantum cluster algebras from surfaces. Consequently, the positivity follows.

2020 Mathematics Subject Classification: 16F30

Keywords: Twisting systems, Twisted algebras, Geometric algebras, Sklyanin algebras, Elliptic curves

Quantum projective planes finite over their centers and Beilinson algebras

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In noncommutative algebraic geometry, a quantum polynomial algebra defined by Artin and Schelter [1] is a basic and important research object, which is a noncommutative analogue of a commutative polynomial algebra. Also, Artin and Schelter [1] gave the classifications of low dimensional quantum polynomial algebras. Moreover, Artin, Tate and Van den Bergh [2] found a nice correspondence between 3-dimensional quantum polynomial algebras and geometric pair (E, σ), where E is the projective plane or a cubic divisor in the projective plane, and σ is the automorphism of E. So, this result allows us to write a 3-dimensional quantum polynomial algebra A as the form $A = A(E, \sigma)$. For a 3-dimensional quantum polynomial algebra $A = A(E, \sigma)$, Artin, Tate and Van den Bergh [3] showed that A is finite over its center if and only if the order $|\sigma|$ of σ is finite. For a 3-dimensional quantum polynomial algebra $A = A(E, \sigma)$ with the Nakayama automorphism ν of A, the author and Mori [7] proved that the order $|\nu^*\sigma^3|$ of $\nu^*\sigma^3$ is finite if and only if the norm $||\sigma||$ of σ introduced by Mori [9] is finite if and only if the noncommutative projective plane in the sense Artin and Zhang [4] is finite over its center. In [6], for a 3-dimensional quantum polynomial algebra A of Type S′, the author prove that the following conditions are equivalent; (1) the noncommutative projective plane is finite over its center. (2) The Beilinson algebra of A is 2-representation tame in the sense of Herschend-Iyama-Oppermann [5]. (3) The isomorphism classes of simple 2-regular modules over the Beilinson algebra of A are parametrized by the projective plane. Note that, this result holds for a Type S by Mori [9]. The Beilinson algebra of A is introduced by Minamoto and Mori [8], which is a typical example of 2-representation infinite algebra.

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2020 Mathematics Subject Classification: 16W50, 16S37, 16D90, 16E65, 16G10, 16G60

Keywords: Quantum polynomial algebras, Geometric algebras, Quantum projective planes, Calabi-Yau algebras, Beilinson algebras, Regular modules

Elliptic algebras

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In 1989, Feigin and Odesskii introduced a family of noncommutative graded algebras $Q_{n,k}(E,\tau)$ called elliptic algebras, which are parametrized by an elliptic curve E, a point $\tau \in E$, and coprime integers $n > k \geq 1$. The algebra $Q_{n,k}(E,\tau)$ is generated by n variables in degree 1 and its relations are defined in terms of Belavin's elliptic solution to the quantum Yang-Baxter equation with spectral parameter.

When $k = 1$, the algebras $Q_{n,1}(E,\tau)$ are called (higher dimensional) Sklyanin algebras in honor of Sklyanin's discovery of $Q_{4,1}(E,\tau)$ in 1982. Sklyanin algebras have been widely studied and recognized as important examples of Artin-Schelter regular algebras. Although Feigin and Odesskii proved and claimed a number of remarkable results on $Q_{n,k}(E,\tau)$ in their series of papers, there are still many things that are only known for $Q_{n,1}(E,\tau)$, but not known for general $Q_{n,k}(E,\tau)$.

For example, it has long been expected that the $Q_{n,k}(E,\tau)$ has the same Hilbert series as the polynomial ring in n variables. This was originally claimed by Feigin and Odesskii, and proved by Tate and Van den Bergh when $k = 1$. One of our main results is that $Q_{n,k}(E,\tau)$ has the same Hilbert series as the polynomial ring in n variables when τ is not a torsion point.

In this talk, starting with motivation and background in noncommutative algebraic geometry, I will explain some properties of Feigin-Odesskii's algebras and how they are obtained from the quantum Yang-Baxter equation. This talk is based on joint work with Alex Chirvasitu and S. Paul Smith.

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2020 Mathematics Subject Classification: 14A22 (Primary), 16S38, 16W50, 17B37, 14H52 (Secondary)

Keywords: Elliptic algebra, quantum Yang-Baxter equation, Sklyanin algebra, Koszul algebra, Artin-Schelter regular algebra

A construction of model structures

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In this talk, we show a method to construct model structures from two weak factorization systems. We then construct a flat model structure on the category of additive functors from a preadditive category satisfying certain conditions to the module category, whose homotopy category is the $\mathcal{Q}$-shaped derived category introduced by Holm and Jørgensen. This talk is based on a joint work with Zhenxing Di, Liping Li and Yajun Ma.

2020 Mathematics Subject Classification:

Keywords:

Ring-theoretic properties in a certain pullback

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Let D be an integral domain with quotient field K, X be an indeterminate over D, $K[X]$ be the polynomial ring over K, $n \geq 2$ be an integer, $K[\theta] = K[X]/(X^n)$, where $\theta := X + (X^n)$, and $R_n := D + \theta K[\theta]$, i.e., $R_n = \{f + (X^n) \in K[X]/(X^n) \mid f(0) \in D\}$. Then R_n is a subring of $K[\theta]$ with total quotient ring $K[\theta]$. In this talk, we will present some ring-theoretic properties of R_n, focusing on Prüfer rings and coherent rings ([1,2]). For example, we show when R_n is a Prufer ring, a Bezout ring, a GCD ring, a coherent ring, or a weakly factorial ring.

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2020 Mathematics Subject Classification: 13A15, 13F05

Keywords: pullback, Prufer ring, Bezout ring, GCD ring, coherent ring, weakly factorial ring

Crystal base of the negative half of the quantum superalgebra for $\mathfrak{gl}_{m|n}$

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The theory of crystal base was introduced by Kashiwara, and it has been one of the most powerful methods in the study of representations of quantum groups. In this talk, we introduce a crystal base of $U_q(\mathfrak{gl}_{m|n})^-$ the negative half of the quantum group associated with a general linear Lie superalgebra $\mathfrak{gl}_{m|n}$, and give a combinatorial description of the associated crystal. This is joint work with Il-Seung Jang and Akito Uruno.

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2020 Mathematics Subject Classification: 17B37

Keywords: crystal bases, quantum superalgebra, general linear Lie superalgebra

Chains of idempotents and bounded generation of $\mathbf{SL}_2$

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We describe interesting connections between association chains of idempotents and bounded generation of SL_2 by elementary matrices. This enables us to identify many classes of matrix rings that have universal bounds on the length of association chains. Consequently, many matrix rings have the property that every regular element is special clean. We also improve the usual criterion for checking perspectivity via association chains, and pose some open problems. This is joint work with Dinesh Khurana and Xavier Mary.

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2020 Mathematics Subject Classification: 16U40, 11A05, 13F07, 16U60, 16U99

Keywords: associate idempotents, bounded generation of SL_2, stable range, terminating division chains

On subalgebras of matrix algebra satisfying some polynomial identity

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A subalgebra of the full matrix algebra $\mathrm{M}_n(K)$, K a field, satisfying the identity

$$[x_1,y_1]\,[x_2,y_2]\cdots[x_q,y_q]=0$$

is called a D_q subalgebra of $\mathrm{M}_n(K)$.

We describe explicitly, up to conjugation, the structure of maximal D_q subalgebras of $\mathrm{M}_n(K)$ as block triangular subalgebras of $\mathrm{M}_n(K)$ with maximal commutative diagonal blocks. The sizes of the diagonal blocks are shown to play critical in deciding when two maximal D_q subalgebras of $M_n(K)$ are isomorphic. In case K is algebraically closed, we invoke Jacobson's characterization of maximal commutative subalgebras of $\mathrm{M}_n(K)$ with maximum (K-)dimension to obtain a complete classification of maximal D_q subalgebras of $\mathrm{M}_n(K)$ which are conjugates of block triangular subalgebras of $\mathrm{M}_n(K)$ with commutative diagonal blocks of maximum dimension.

2020 Mathematics Subject Classification:

Keywords:

Contributed talks

$R[X]_A$ of zero-dimensional reduced rings

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Let R be a commutative ring with identity and let $R[X]$ be the polynomial ring over R. Consider the following two subsets of $R[X]$:

$$N := \{f \in R[X] \mid c(f) = R\} \text{ and}$$
$$U := \{f \in R[X] \mid f \text{ is a monic polynomial}\}.$$

Then N and U are multiplicative subset of $R[X]$, so we obtain the rings $R[X]_N$ and $R[X]_U$, which are called the *Nagata ring* of R and *Serre's conjecture ring* of R respectively. The Nagata rings and the Serre's conjecture rings have been researched actively.

In this talk, we investigate the new subring $R[X]_A$ of Nagata ring and Serre's conjecture ring and examine following problems:

(1) If $R[X]_A$ is a PIR, then is R a PIR? What about the converse?

(2) If $R[X]_A$ is an arithmetical ring, then is R an arithmetical ring? What about the converse?

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2020 Mathematics Subject Classification: 13A15

Keywords: Nagata ring, Serre's conjecture ring, zero-dimensional reduced ring, principal ideal ring, arithmetical ring, Prüfer ring

On the modules having proper S-essential submodules

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In this talk, we study S-essential submodules which are a generalization of essential submodule of modules. Besides giving many examples and properties of S-essential submodules, we generalize some results on essential submodules to S-essential submodules. Finally, we can show that for a nonzero right R-module M such that $Ann_R(m) \cap S = \varnothing$ for all $m \in M - \{0\}$, M has a proper S-essential submodule if and only if a right $R/Ann_R(M)$-module M has a proper $\overline{S}$-essential submodule where $\overline{S} = \{s + Ann_R(M) | s \in S\}$. This is joint work with Poramate sangchan.

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2020 Mathematics Subject Classification: 13A15, 16P40

Keywords: multiplicatively closed subset, essential submodule, S-essential submodule, essential monomorphism, S-essential monomorphism

A new method to quantify Integrable Systems

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We discuss a new method of quantifying classical Integrable Systems via the example of the Toda hierarchy. We first lift these systems to a free associative setting and look for the right quantifying ideals in these noncommutative algebras. We will see that in the case of the Toda hierarchy we first retrieve a well-known deformation quantization coming from R matrices, but that we also find a new quantum integrable system which so far does not fit in the deformation quantization picture. This is a joint work with A. Mikhailov (Leeds) and J.P. Wang (Kent, UK).

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2020 Mathematics Subject Classification: 37K06, 37K10

Keywords: Integrable Systems of differential-difference equations, quantum Integrable Systems, Yang-Baxter equation, Toda hierarchy

Rings and radicals related to n-primariness

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An ideal I of a ring R is called *n-primary* (resp., *T-primary*) provided that $AB \subseteq I$ for ideals A, B of R implies that $(A+I)/I$ or $(B+I)/I$ is nil of index n (resp., $(A+I)/I$ or $(B+I)/I$ is nil) in R/I, where $n \geq 1$. It is proved that for a proper ideal I of a principal ideal domain R, I is T-primary if and only if I is of the form p^kR for some prime element p and $k \geq 1$ if and only if I is 2-primary, through which we study the structure of matrices over principal ideal domains. We prove that for a T-primary ideal I of a ring R, R/I is prime when the Wedderburn of R/I is zero. In addition we provide a method of constructing strictly descending chain of n-primary radicals from any domain, where the *n-primary radical* of a ring R means the intersection of all the n-primary ideals of R. This is joint work with Yang Lee.

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2020 Mathematics Subject Classification: 16D25, 16S36, 16U80

Keywords: n-priamry ring, T-primary ring, n-priamry radical, T-primary radical, prime ring, polynomial ring, matrix ring

Modules of 0-Hecke algebras from posets

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In this talk, our main focus is on exploring a specific class of mathematical objects called right 0-Hecke modules, which arise from partially ordered sets. These modules, also known as poset modules, were introduced by Duchamp, Hivert, and Thibon. Their work involved investigating the intriguing algebraic and combinatorial properties of these modules.

We specifically consider the category of the full subcategory of right 0-Hecke modules, where the objects are direct sums of finitely many isomorphic copies of poset modules. Our first result establishes that the direct sum of the Grothendieck groups of the poset module category is isomorphic to the Hopf algebra of quasisymmetric functions. Furthermore, we demonstrate that every right weak Bruhat interval module can be viewed as a poset module. By introducing a certain functor, we also establish that every left weak Bruhat interval module can be viewed as a poset module. This is joint work with Young-Hun Kim and Young-Tak Oh.

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2020 Mathematics Subject Classification: 20C08, 05E05, 05E10

Keywords: 0-Hecke algebra, quasisymmetric function, partially ordered set

On some classes of subalgebras of Leavitt path algebras

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The history of Leavitt path algebras dates back to the 1960s, when William G. Leavitt posed a question regarding the existence of R rings that satisfy the equality $R^i = R^j$ as right-hand modules over R. The concept of Leavitt path algebras was introduced between 2005 and 2007. This construction was developed to algebraically represent combinatorial objects associated with Cuntz-Krieger algebras and C^*-algebras. The study of Leavitt path algebras is of interest to both algebraists and functional analysts, particularly those working with C^*-algebras. The flexible nature of Leavitt path algebra construction allows for the generation of numerous examples of algebras with specific properties.

An important special case of Leavitt path algebras is the class of matrix algebras $M_n(F)$ over a field F, where n is a natural number or infinity (infinite-size matrices have a countable number of rows and columns with a finite number of non-zero elements in each). In my presentation, I will recall definition of Leavitt path algebras. Based on this, I will demonstrate a construction of maximal commutative subalgebras of Leavitt path algebras and their connections with well-known examples of maximal commutative subalgebras of matrix algebras over a field.

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2020 Mathematics Subject Classification:

Keywords:

On the finiteness of certain factorization invariants

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To study factorization problems in rings and monoids, we introduce a concept of minimal factorization that helps to counter blow-up phenomena triggered, for example, by the presence of non-trivial idempotent factors. As it happens in the classical theory of factorization, it is possible to associate these minimal factorizations with an assortment of invariants that measure their non-uniqueness. In this talk we discuss some finiteness results for such invariants, focusing in particular on minimal elasticity. This is joint work with Salvatore Tringali.

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2020 Mathematics Subject Classification: 20M13, 13A05, 08A50, 20M05

Keywords: minimal factorizations, elasticity, sets of lengths, irreducibles, preorders

The Centralizing Maps on Modules by using Idealization of Modules

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Let R be a commutative ring, and let M be an R-module. Nagata introduced the idealization $R(+)M$ of M. $R(+)M = R \oplus M$ with addition $(r_1,m_1)+(r_2,m_2) = (r_1+r_2,m_1+m_2)$ and multiplication $(r_1,m_1)(r_2,m_2) = (r_1r_2,r_1m_2+r_2m_1)$ is a ring. It is called the idealization of M. The idealization can be used to extend results about ideal to modules. A mapping f of R into itself is called centralizing if $[f(x),x] \in Z(R)$ holds for all $x \in R$. In this paper, we define the centralizing maps maps on module M over ring R. In this construction, we use the concept of idealization of modules. Furthermore, we give some examples of the centralizing maps of modules and investigate the characterization of the centralizing maps of modules. This is joint work with Indah Emilia Wijayanti, Ahamd Faisol, Nikken Prima Puspita and Ida Kurnia Waliyanti.

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2020 Mathematics Subject Classification: 16D10, 16S20

Keywords: Idealization, Module, centralizing

Powers of ghost ideals

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In this talk, the theory of infinite powers of ghost ideals is presented. A key tool developed in this theory is an ideal version of Eklof's Lemma. This theory is used to study Generalized Generating Hypothesis. In particular, (1) it is used to show a dual of a result of Xu: if the class of pure projective right R-modules is closed under extensions, then every FP-projective right R-module is pure projective; and (2) it is used to study the ghost ideal in the category of complexes. This is joint with S. Estrada, I. Herzog, and S. Odabasi.

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2020 Mathematics Subject Classification: 18E10, 18G15, 18G25, 16G70, 16N20

Keywords: ghost ideals, Eklof's Lemma, Generalized Generating Hypothesis, FP-projective modules, pure projective modules

A Duality Theorem for Monoidal Hom Hopf Module Algebras

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We prove a duality theorem for (possibly) infinite dimensional monoidal Hom Hopf algebras (H,α) over k. In the infinite dimensional case, H^* is too big and we replace H^* by finite dual H° of (H,α) or, more generally, a monoidal Hom Hopf subalgebra U of H°. It is also necessary to assume that H and U have bijective antipodes. Let (A,β) be a (H,α) Hom module algebra that the action of (H,α) on (A,β) is locally finite in a sense appropriate to the choice of U, and that the action of $H\sharp U$ satisfies a certain RL-condition. Under these conditions, we we have a Hom algebra isomorphism $(A\sharp H)\sharp U \cong A \otimes (H\sharp U)$.

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2020 Mathematics Subject Classification: 16A24

Keywords: monoidal Hom-Hopf algebras, Hom-Hopf module algebra, Hom-smash product, duality theorem

The absorption laws and the reverse order laws of two kinds of generalized core inverses

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As an extension of the core inverse, in 2014, Manjunatha Prasad and Mohana, Malik and Thome introduced, respectively, the core-EP inverse and the DMP inverse which both exist for arbitrary square complex matrices. These two generalized core inverses can be discussed in a ring with involution. In this talk, we will give necessary and sufficient conditions for which the absorption laws and the reverse order laws of two generalized core inverses hold in a ring with involution.

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2020 Mathematics Subject Classification: 16W10, 15A09

Keywords: absorption law, reverse order law, core-EP inverse, DMP inverse, involutory ring

(m,n)-Jordan $*$-Centralizers and Their Generalizations over $*$-Prime and $*$-Semi-prime Rings

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In this presentation, we introduce the novel concept of (m,n)-Jordan $*$-centralizer over $*$-rings, inspired by the existing definitions of (m,n)-Jordan centralizer and $*$-centralizer. We also propose the concept of generalized (m,n)-Jordan $*$-centralizer, which combines the notions of generalized (m,n)-Jordan centralizer and $*$-centralizer. Our focus is on studying these concepts over $*$-prime and $*$-semi-prime rings, analyzing their properties and behavior. Our results provide a useful framework for further exploration of (m,n)-Jordan $*$-centralizers and their generalizations.

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2020 Mathematics Subject Classification: 16W25, 16S50, 16S60, 16W50

Keywords: (m,n)-Jordan centralizer, $*$-centralizer, (m,n)-Jordan $*$-centralizer, generalized (m,n)-Jordan $*$-centralizer, $*$-prime rings, $*$-semi-prime rings

Morita contexts for Hopf coquasi-module algebra

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Let H be a Hopf coquasigroup, and A be a left coquasi H- module algebra, we show that there is a Morita context related the subalgebra A^H and the smash product $A\sharp H$ by constructing the associated bimodules ${}_{A^H}A_{A\sharp H}$, ${}_{A\sharp H}A_{A^H}$ and by constructing the corresponding bimodular maps. Furthermore, some conditions for the Morita equivalence are given. This is joint work with Shuan Hong Wang.

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2020 Mathematics Subject Classification: 16T05, 16W99

Keywords: Hopf coquasigroup, Morita context, module algebra

Torsion lattice of GLS path algebra and Cambrian lattice

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In representation theory of algebra, a torsion class refers to a subcategory of the category of finitely generated modules that is closed under extensions and quotients. The collection of all such subcategories forms a partially ordered set, called the torsion lattice. On the other hand, a Cambrian lattice is also a lattice that arises from a particular set of elements in the Weyl group associated with a root system. Both of these lattice structures are highly versatile and appear in various fields in different forms. In 2006, Ingalls and Thomas showed that the torsion lattice arising from the path algebra associated with ADE-type Dynkin quiver and the Cambrian lattice arising from the same Dynkin quiver coincide, and since then, there have been many studies on the relationship between these two lattices. In this talk, we explain the extension of the isomorphism between the lattices discovered by Ingalls and Thomas to Dynkin graphs that are not simply-laced. Specifically, we discuss the isomorphism between the GLS path algebra, defined by Geiß-Leclerc-Schröer, and the Cambrian lattice.

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2020 Mathematics Subject Classification: 13F60, 16G20, 17B22

Keywords: Cambrian lattice, torsion lattice, Dynkin diagram

Quotient singularities, higher hereditary algebras, and non-commutative regular algebras

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We study the relationship of representation theories of three different types of rings: (1) commutative Gorenstein rings R, (2) finite dimensional algebras A, and (3) non-commutative regular algebras Γ (or Artin-Schelter regular algebras). We do so by looking at triangulated categories naturally associated to each of them: the singularity category of R, the derived category of A, and the derived category of the non-commutative projective scheme of Γ. Given a certain class of commutative Gorenstein ring R, we construct a tilting object in its graded singularity category, whose endomorphism ring A turns out to be higher representation infinite. It consequently yields a non-commutative regular algebra Γ. The construction from A to Γ can be explained as a certain variation of the Calabi-Yau completion.

2020 Mathematics Subject Classification: 13C14, 16G10, 16S38, 18G80

Keywords: Quotient singularity, higher representation infinite algebra, Calabi-Yau algebra, Calabi-Yau completion

Combinatorial superalgebras and their superproducts

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Supersymmetry is an important concept in mathematical physics and the treatment of structures such as Clifford algebras [2]. When applied to linear spaces, it entails a direct sum decomposition with two homogeneous summands, respectively described as "even" and "odd". In a combinatorial or set-theoretical context, it becomes simpler: just a disjoint union decomposition into two uniands, again described as even and odd. A set decomposed in this way is called a "superset".

When (linear) supersymmetry is applied to real Clifford algebras, the Clifford algebra of the direct sum of two real quadratic spaces emerges as the superalgebra tensor product $\widehat{\otimes}$ of the respective Clifford algebras of the summands [1].

We present a combinatorial version of the superalgebra tensor product which will be applicable equally to groups. This is joint work with J.D.H. Smith.

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2020 Mathematics Subject Classification: 20N05, 17A70, 20C15

Keywords: superalgebra, superalgebra tensor product, superproduct

Some ring-theoretic properties of a local log-regular ring

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Toric varieties (toric rings) are algebraic varieties (commutative rings) formulated by combinatorial information and have played important roles in algebraic geometry and commutative ring theory. On the other hand, establishing toric theory without bases is much important for studying arithmetic geometry and commutative ring theory in mixed characteristic. To establish the theory, Kazuya Kato introduced local log-regular rings. A local log-regular ring is a commutative ring with an additional structure defined by a homomorphism of monoids. This class of rings has the structure theorem such as Cohen's structure theorem. This theorem suggests that this class has similarities to the class of toric rings. From this point of view, we obtain some ring-theoretic properties (the structure of canonical modules and some finiteness of the divisor class group) of a local log-regular ring. In this talk, we introduce these results.

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2020 Mathematics Subject Classification: 13C20, 14A21

Keywords: Logarithmic geometry, Local log-regular rings, canonical modules, divisor class groups

On the relation between perfectoidisation and p-root closure

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Perfectoidization was introduced by Bhatt-Scholze as a mixed characteristic analogue of of the perfection of rings of positive characteristic. On the other hand, the notion of p-root closure was introduced into commutative ring theory by Roberts. This is a simple closure operation that existed before the appearance of perfectoid rings, and is closely related to the structure of the Fontaine ring (now often called a tilting). In this talk, I will discuss the relationship between these two operations. In particular, I will show that a certain perfectoidization coincides with the p-adic completion of the p-root closure.

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2020 Mathematics Subject Classification: 14G45, 46J05

Keywords: Perfectoid rings, Perfectoidization, p-root closure

Hochschild homology dimension and a class of Hochschild extension algebras of truncated quiver algebras

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In this talk, we show that higher Hochschild homology groups do not vanish for a class of Hochschild extension algebras of truncated quiver algebras by the standard duality module. Let $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$ and A a finite dimensional algebra over a field K. The Hochschild homology dimension $\mathrm{HH}\dim A$ of A is defined by $\mathrm{HH}\dim A = \inf\{n \in \mathbb{N}_0 \mid \mathrm{HH}_n(A) = 0\}$. Han conjectured that if $\mathrm{HH}\dim A$ is finite, then the global dimension of A is finite, in [2]. This is the homology version of Happel's question. After that, many classes of algebras which have infinite Hochschild homology dimension were discovered. In [1], Bergh and Madsen showed that the Hochschild homology dimension of trivial extension algebras by the standard duality module are infinite for selfinjective algebras, local algebras and graded algebras. We focus on truncated quiver algebras, and we consider Hochschild extension algebras by the standard duality module and their Hochschild homology dimension. Hochschild extension algebras are given by 2-cocycles $A \times A \to DA = \mathrm{Hom}_K(A, K)$. In particular, the trivial extension algebra is the Hochschild extension algebra by the zero map. It is well known that there is a one-to-one correspondence between the set of the equivalent classes of Hochschild extension and the second Hochschild cohomology group $H^2(A, DA)$ of A with coefficients in DA which is isomorphic to the second Hochschild homology group $\mathrm{HH}_2(A)$ of A as K-modules. Let $m \geq 2$, Q a finite quiver, R_Q the arrow ideal of KQ and $B = KQ/R_Q^m$. For each $n \geq 0$, the n-th Hochschild homology group $\mathrm{HH}_n(B)$ is $\mathbb{N}_0$-graded, and its l part $\mathrm{HH}_{n,l}(B)$ was computed in [3]. In particular, $\mathrm{HH}_2(B) = \oplus_{l=m}^{2m-1} \mathrm{HH}_{2,l}(B)$. In this talk, we show that for any 2-cocycle $\alpha : B \times B \to DB$ corresponding to an element in $\oplus_{l=m}^{2m-2} \mathrm{HH}_{2,l}(B)$ or $\mathrm{HH}_{2,2m-1}(B)$, the Hochschild homology dimension of the Hochschild extension algebra by α is infinite.

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Keywords: Hochschild extension algebras, Hochschild homology, truncated quiver algebras

Tensor products and almost split sequences for group rings

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Let G be a finite group and p a prime number dividing the order of G. Let $(K, \mathcal{O}, k)$ be a p-modular system, namely, K is a complete discrete valuation field of characteristic zero with multiplicative valuation ν, $\mathcal{O}$ is a valuation ring of ν with unique maximal ideal $\pi\mathcal{O}$, and $k = \mathcal{O}/\pi\mathcal{O}$ is the residue class field of $\mathcal{O}$ of characteristic p. We assume that k is algebraically closed. We use R to denote $\mathcal{O}$ or k, and we denote by RG the group ring of G over R. RG-lattices means finite generated right RG-modules which are free as R-modules. If V and W are RG-lattices, $V \otimes_R W$ and $\mathrm{Hom}_R(V, W)$ are RG-lattices with the operations of G given by $(x \otimes y)g = xg \otimes yg$ and $[\varphi g](x) = \varphi(xg^{-1})g$ for all $g \in G,\ x \in V,\ y \in W$and $\varphi \in \mathrm{Hom}_R(V, W)$. Also, R_G denotes the trivial RG-lattice and V^* denotes the dual RG-lattice $\mathrm{Hom}_R(V, R_G)$ of V.

For a non-projective indecomposable RG-lattice U, we denote by $\mathscr{A}(U)$ the almost split sequence terminating in U: $0 \longrightarrow \tau U \longrightarrow m(U) \longrightarrow U \longrightarrow 0$. Applying $V \otimes_R -$ to $\mathscr{A}(U)$ for an indecomposable RG-lattice V, we get the exact sequence

$$V \otimes_R \mathscr{A}(U) : 0 \longrightarrow V \otimes_R \tau U \longrightarrow V \otimes_R m(U) \longrightarrow V \otimes_R U \longrightarrow 0.$$

In the case where $U = R_G$, Auslander and Carlson showed that $V \otimes_R \mathscr{A}(R_G)$ is a direct sum of $\mathscr{A}(V)$ and a split sequence if and only if the multiplicity of the trivial RG-lattice R_G in $V \otimes_R V^*$ is one [1].

In this talk, we consider the tensor product sequence $V \otimes_R \mathscr{A}(Sc(Q))$, where Q is a p-subgroup of G and $Sc(Q)$ is the Scott RG-lattice with vertex Q. Then we see that for an indecomposable RG-lattice V with vertex Q, $V \otimes_R \mathscr{A}(Sc(Q))$ is a direct sum of $\mathscr{A}(V)$ and a split sequence if and only if the multiplicity of $Sc(Q)$ in $V \otimes_R V^*$ is one.

In the case $R = \mathcal{O}$, Knorr introduced the notion of virtually irreducible $\mathcal{O}G$-lattices [3]. We also see that for an indecomposable $\mathcal{O}G$-lattice L with vertex Q, the following conditions (i), (ii) and (iii) are equivalent:

(i) L is virtually irreducible and the $\mathcal{O}$-rank of a Q-source of L is not divisible by p;

(ii) $L \otimes_{\mathcal{O}} \mathscr{A}(Sc(Q))$ is a direct sum of $\mathscr{A}(L)$ and a split sequence;

(iii) The multiplicity of $Sc(Q)$ in $L \otimes_{\mathcal{O}} L^*$ is one [2].

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2020 Mathematics Subject Classification: 16G70, 20C11, 20C20

Keywords: Representations of finite groups, Almost split sequences, Scott lattices

On the Auslander-Reiten conjecture for normal rings

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Auslander and Reiten [3] proposed the generalized Nakayama conjecture, which is rooted in the Nakayama conjecture [8]. In addition, they proposed another conjecture about projectivity of module by vanishing of Ext modules, which is called the Auslander-Reiten conjecture, and proved that this conjecture is true if and only if the generalized Nakayama conjecture is true. This long-standing conjecture is known to hold true for several classes of algebras.

The Auslander-Reiten conjecture remains meaningful for arbitrary commutative noetherian rings for formalization by Auslander, Ding, and Solberg [2]. The conjecture is known as follows: for a commutative noetherian ring R, every finitely generated R-module M such that $\mathrm{Ext}^i_R(M, M \oplus R) = 0$ for all $i \geq 1$ is projective. This conjecture is known to hold if R is a complete intersection [2], or if R is a locally excellent Cohen-Macaulay normal ring containing the field of rational numbers $\mathbb{Q}$ [5], or if R is a Gorenstein normal ring [1]. Recently, Kimura, Otake, and Takahashi [7] proved the conjecture for every Cohen-Macaulay normal ring. Even if R is not Cohen-Macaulay, it is known that R satisfies the conjecture if it is a quotient of a regular local ring and is a normal ring containing $\mathbb{Q}$ [4]. In this talk, we consider the above conjecture over normal rings.

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2020 Mathematics Subject Classification: 13D07

Keywords: Auslander-Reiten conjecture, Ext module, Serre's condition

On support τ-tilting modules over group algebras

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Since τ-tilting theory was introduced by Adachi-Iyama-Reiten in 1, the theory continues to develop rapidly. The one of main themes of the theory is the study of support τ-tilting modules, and today, there are many studies of the modules for various kinds of algebras. In this talk, we consider the support τ-tilting modules for group algebras of finite groups.

Let k be an algebraically closed field of characteristic $p > 0$, $\tilde{G}$ a finite group, G a finite group, M a support τ-tilting kG-modules, and $\tilde{M}$ a support τ-tilting $k\tilde{G}$-module. We consider when $\tilde{M}$ is a support τ-tilting module as kG-module, and when the induced module $k\tilde{G} \otimes_{kG} M$ is a support τ-tilting $k\tilde{G}$-module. Moreover, as an application, we give a feature of vertices of the support τ-tilting modules for group algebras. This is joint work with Ryotaro Koshio.

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2020 Mathematics Subject Classification: 20C20

Keywords: τ-tilting theory, support τ-tilting modules, group algebras, block algebras, induction functors, restriction functors

Hirata separability, partial Galois extension and inner partial group action

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In this talk, we firstly review some Sugano's results on Galois extensions and then present our recent work generalizing Sugano's results in the context of partial group action. More explicitly, we give some equivalent conditions for partial Galois extensions to be Hirata-separable extensions and show that every partial Galois extension with inner partial group action is Hirata-separable.

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2020 Mathematics Subject Classification: 13B05, 16W22

Keywords: partial group action, partial Galois extension, Hirata separable

Structure of right APIP rings

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We study the following two conditions in rings: (i) the right annihilator of some power of any element is an ideal, and (ii) the right annihilator of any nonzero element a contains an ideal generated by some power of any right zero-divisor of the element a. We investigate the structure of rings in relation to these conditions; especially, a ring with the condition (ii) is called right APIP. These conditions are shown to be not right-left symmetric. For a prime two-sided APIP ring R we prove that every element of R is either nilpotent or regular, and that if R is of bounded index of nilpotency then R is a domain. We also provide several interesting examples which delimit the classes of rings related to these properties.

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2020 Mathematics Subject Classification: 16U80

Keywords: Annihilator, right APIP ring, K-ring, nilpotent element, prime ring, Köthe's conjecture, matrix ring

The structure of rings which are symmetric on the set of all idempotents

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Alghazzawi and Leroy studied the structure of subsets satisfying the properties of symmetric and commutatively closed, that is, $abc \in S$ for $a, b, c \in R$ implies $acb \in S$, and $ab \in S$ for $a, b \in R$ implies $ba \in S$, respectively, where S is a subset of a ring R. In this article we discuss the structure of rings which are symmetric on zero (resp., idempotents). Such rings are also called *symmetric* (resp., *I-symmetric*). We first prove that if a polynomial $\sum_{i=0}^{m} a_i x^i$ over a symmetric ring is a unit then a_0 is a unit and a_i is nilpotent for all $i \geq 1$; based on this result, we obtain that for a reduced ring R, the group of all units of the polynomial ring over R coincides with one of R, and that polynomial rings over I-symmetric rings are identity-symmetric. It is proved that for an abelian semiperfect ring R, R is I-symmetric if and only if the units in R form an Abelian group if and only if R is commutative. It is also proved that for an I-symmetric ring R, R is π-regular if and only if $R/J(R)$ is a commutative regular ring and $J(R)$ is nil, where $J(R)$ is the Jacobson radical of R. This is joint work with Yang Lee.

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2020 Mathematics Subject Classification: 16U80, 16U60, 16D25, 16S50, 16N40

Keywords: I-symmetric ring, symmetric ring, reversible ring, idempotent

Dirac reductions and classical W-algebras

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In this talk, I explain the Dirac reduction originated from classical mechanics in the realm of Poisson algebras. Next, I explain the modification of this reduction and how to relate the structures of classical W-algebras in the perspective of the modified Dirac reduction. This is joint work with Arime Song and Uhi Rinn Suh.

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2020 Mathematics Subject Classification: 17B69, 81R10

Keywords: Poisson algebra, Dirac reduction, Classical W-algebra

On polyform modules

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The rational extension of a ring is developed by R.E. Johnson in 1951 and Y. Utumi in 1956 in order to generalize the quotients ring theory. In 1958, G.D. Findlay and J. Lambek defined a relationship between three modules over a ring R, $A_R \leq B_R(C_R)$, which means that A_R is a relatively dense submodule of B_R to C_R. Recently, in [1], as an application of the relatively dense property, the equivalent condition for the rational hull of the finite direct sum of modules to be the direct sum of the rational hulls of those modules is shown. Polyform modules, known as non-M-singular modules, are studied by R. Wisbauer (1981) and J. M. Zelmanowitz (1986). Recall that a right R-module is said to be *polyform* if every essential submodule of M is a dense submodule. It is known that the endomorphism ring of the quasi-injective hull of a polyform module is von Neumann regular. See [2, Section 11] for more properties of polyform modules.

In this talk, we provide polyform modules and introduce the notion of relatively polyform modules over a ring R. Some new characterizations and properties of polyform modules are obtained. For a retractable polyform module M_R, it is proved that the maximal right ring of quotients of $\text{End}_R(M)$ is $\text{End}_R(\widetilde{E}(M))$, where $\widetilde{E}(M)$ is the rational hull of M. Moreover, it is proved that the class of all polyform right R-modules is closed under submodules and rational extensions. This is joint work with Xiaoxiang Zhang and Nguyen Khanh Tung.

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2020 Mathematics Subject Classification: 16D25, 16SD50, 16D80

Keywords: rational hull, (relatively) dense submodule, (relatively) polyform module, $\mathcal{K}$-nonsingular module

Super duality for quantum affine algebras of type A

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Super duality is a novel and powerful method in the representation theory of Lie superalgebras. In this talk, we introduce a new approach to finite-dimensional representations of the quantum group associated with the affine Lie superalgebra $L\mathfrak{gl}_{M|N} = \mathbb{C}[t, t^{1}] \otimes \mathfrak{gl}_{M|N}$ $(M \neq N)$. Motivated from super duality, we explain how the representations of the quantum group of $L\mathfrak{gl}_{M|N}$ are directly related to those of the quantum group of $L\mathfrak{gl}_n$, using an exact monoidal functor called truncation. This is joint work with Jae-Hoon Kwon.

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2020 Mathematics Subject Classification: 17B10, 17B37

Keywords: Super duality, Lie superalgebra, quantum affine algebra

Recollements and n-cluster tilting subcategories

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In this paper, we study the relationship among three n-cluster tilting subcategories of triangulated categories in a recollement. Let (D', D, D'') be a recollement of triangulated categories. We construct an ncluster tilting subcategory of D by gluing together n-cluster tilting subcategories of D' and D'' under certain conditions. On the other hand, we prove that an n-cluster tilting subcategory of D can induce n-cluster tilting subcategories of D' and D''. Using these two results, we give a one-to-one correspondence between some n-cluster tilting subcategories of D and pairs of n-cluster tilting subcategories of D' and D''. Furthermore, we obtain a recollement of abelian categories from a recollement of triangulated categories. This is joint work with Xiaoxiang Zhang and Yukun Zhou.

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2020 Mathematics Subject Classification: 18G80, 18A40

Keywords: Recollement, n-cluster tilting subcategory, triangulated category, abelian category

Homological properties of modules over trivial ring extensions

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It is well known that the class of trivial ring extensions covers Morita context rings with bimodule homomorphisms zero, particularly, covers formal triangular matrix rings. We give some homological properties of trivial ring extensions and describe explicitly some important classes of modules over a trivial ring extension.

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2020 Mathematics Subject Classification: 16D40, 16D50

Keywords: Trivial extension, pure projective module, pure injective module

Classification of twisted algebras of 3-dimensional Sklyanin algebras

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The notion of twisting system was introduced by Zhang [3]. A twisting system is called algebraic if it is given by some graded algebra automorphism. One of the main applications of a twisting system is that the graded module category of twisted algebra is equivalent to that of the original graded algebra. Although a twisting system is useful, it is often difficult to construct non-algebraic twisting systems even if a graded algebra is given by explicit generators and defining relations. A geometric algebra $A = \mathcal{A}(E,\sigma)$ introduced by Mori [2] is a quadratic algebra which determines and is determined by the pair (E,σ) where E is a projective scheme and $\sigma \in \mathrm{Aut}_k E$. A 3-dimensional Sklyanin algebra is a geometric algebra $A = \mathcal{A}(E,\sigma)$ where E is an elliptic curve in the projective plane $\mathbb{P}^2$ and σ is a translation by some point of E. One of our main results is that we provide a geometric condition about whether a twisting system on A is algebraic. Using this condition, we can construct a lot of non-algebraic twisting systems on A. As an application, we classify twisted algebras of 3-dimensional Sklyanin algebras up to graded algebra isomorphism.

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2020 Mathematics Subject Classification: 14A22, 16S38, 16W50

Keywords: Twisting systems, Twisted algebras, Geometric algebras, Sklyanin algebras, Elliptic curves

Classification of locally free sheaf bimodules of rank 2 over the projective line

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This talk is based on the joint work with Shinnosuke Okawa and Kazushi Ueda. To complete the fundamental project in noncommutative algebraic geometry, namely, Artin's project on the classification of noncommutative surfaces, it is essential to classify quantum ruled surfaces. In this talk, we will explain some strategies to classify quantum Hirzebruch surfaces, which are defined to be quantum ruled surfaces over the projective line. It reduces to classify locally free sheaf bimodules of rank 2 over the projective line. Here, (commutative) algebraic geometry and (Cohen-Macaulay) representation theory are basic tools.

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2020 Mathematics Subject Classification: 18E15, 14D15

Keywords: Noncommutative algebraic geometry, quantum Hirzebruch surfaces, locally free sheaf bimodules

Retracts of Laurent polynomial rings

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Let R be an integral domain and A, B be R-algebras. A is called a retract of B if there exists an ideal I of B such that B is the direct sum of A and I. In this talk, we consider retracts of Laurent polynomial rings over R in n variables. We show that such retract is isomorphic to a Laurent polynomial ring over R.

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2020 Mathematics Subject Classification: Primary: 13B25, Secondary: 14A05

Keywords: retract, Laurent polynomial ring, polynomial ring

K_0 of weak Waldhausen extriangulated categories

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A Waldhausen category is a generalization of an exact category. It is used to define the K-theory on a wide class of categories which generalizes Quillen's K-theory. In this talk, focusing on the Grothendieck groups, we modify the axiom of the Waldhausen structure so that it matches better with extriangulated categories. It enables us to define an abelian group $K_0(\mathsf{C})$ of a weak Waldhausen category C which generalizes the usual Grothendieck group of an extriangulated category. As one might expect, it behaves nicely in the context of Quillen's localization and resolution theorems. We obtain two applications: the first one generalizes exact sequences of the Grothendieck groups associated with the Serre/Verdier localization to some types of "one-sided exact" localizations; the second one reveals close relations between Quillen's theorems and Palu's index. This is joint work with Amit Shah.

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2020 Mathematics Subject Classification: 16E20, 18G80, 18E35

Keywords: Extriangulated category, Waldhausen category, Grothendieck group, localization

Factorization in the (composite) Hurwitz rings

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Let R be a commutative ring with identity and $H(R)$ be the set of formal power series over R. Addition on $H(R)$ is defined termwise. A multiplication, called $*$-product, on $H(R)$ is defined as follows: For $f = \sum_{n=0}^{\infty} a_n x^n, g = \sum_{n=0}^{\infty} b_n x^n \in H(R)$,

$$f * g = \sum_{n=0}^{\infty} c_n x^n, \;\; c_n = \sum_{k=0}^{n} \binom{n}{k} a_k b_{n-k}, \;\text{ where } \binom{n}{k} = \frac{n!}{(n-k)!k!}.$$

Then $H(R)$ is a commutative ring with identity under these two operations [1] and then call $H(R)$ the ring of Hurwitz series over R [2]. The ring $h(R)$ of Hurwitz polynomials over R is the subring of $H(R)$ consisting of polynomials $\sum_{k=0}^{n} a_k x^k$. We simply call $H(R)$ and $h(R)$ Hurwitz rings.

For an extension $A \subseteq B$ of commutative rings with identity, let

- $H(A,B) = \{f \in H(B) |$ the constant term of f belongs to $A\}$, and
- $h(A,B) = \{f \in h(B) |$ the constant term of f belongs to $A\}$.

Then these are commutative rings with identity. We call $H(A,B)$ and $h(A,B)$ composite Hurwitz rings.

In this talk, we introduce several factorization properties (UFD, BFD, and FFD) on (composite) Hurwitz rings.

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2020 Mathematics Subject Classification: 13B25, 13F15

Keywords: Composite Hurwitz series ring, composite Hurwitz polynomial ring, UFD, bounded factorization domain, finite factorization domain

The n-torsionfreeness of syzygies of the residue field

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The notion of n-torsionfree modules was introduced by Auslander and Bridger [1] as a natural generalization of the notion of torsionfree modules over integral domains. The n-torsionfreeness of syzygies plays an important role in Auslander's approximation theory [1, 2]. As a dual theory of approximation theories, the existence of finite projective hulls has been well-studied [2, 4, 5, 6]. We will see that the existence of finite projective hulls is closely related to the $(n+1)$-torsionfreeness of n-syzygies. As an application, we consider the n-torsionfreeness of syzygies of the residue field k over a local ring R. Let $t = \operatorname{depth} R$ and $\Omega^t k$ be the tth syzygy of the residue field k. Dey and Takahashi [3] proved that $\Omega^t k$ is $(t+1)$-torsionfree, and it is a $(t+2)$nd syzygy if and only if the local ring R has type one. Motivated by their results, we study higher torsionfreeness of the syzygy $\Omega^t k$.

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2020 Mathematics Subject Classification: 13D02, 13D07

Keywords: n-torsionfree module, n-syzygy module, n-spherical module, projective dimension, Gorenstein ring

The first Hilbert coefficient and the reduction number of stretched ideals

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The homological property of the associated graded ring of an ideal is an important problem in commutative algebra. In this talk we explore the structure of the associated graded ring of stretched $\mathfrak{m}$-primary ideals in the case where the reduction number attains almost minimal value in a Cohen-Macaulay local ring $(A, \mathfrak{m})$. In the second half of this talk, We also present the structure of stretched $\mathfrak{m}$-primary ideals with small first Hilbert coefficient.

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2020 Mathematics Subject Classification: 13H10, 13D40

Keywords: commutative ring, stretched local ring, stretched ideal, Cohen-Macaulay local ring, associated graded ring, Hilbert function, Hilbert coefficient

Characterizations of standard derived equivalences of diagrams of dg categories and their gluings

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A diagram consisting of differential graded (dg for short) categories and dg functors is formulated as a colax functor X from a small category I to the 2-category $kdgCat$ of small dg categories, dg functors and dg natural transformations for a fixed commutative ring k. If I is a group regarded as a category with only one object $*$, then X is nothing but a colax action of the group I on the dg category $X(*)$. In this sense, this X can be regarded as a generalization of a dg category with a colax action of a group. We define a notion of standard derived equivalence between such colax functors by generalizing the corresponding notion between dg categories with a group action. Our first main result gives some characterizations of this notion, one of which is given in terms of generalized versions of a tilting object and a quasi-equivalence. On the other hand, for such a colax functor X, the dg categories $X(i)$ with i objects of I can be glued together to have a single dg category $Gr(X)$, called the Grothendieck construction of X. Our second main result insists that for such colax functors X and X', the Grothendieck construction $Gr(X')$ is derived equivalent to $Gr(X)$ if there exists a standard derived equivalence from X' to X. These are new even for dg categories with a group action. In particular, the second result gives a new tool to show the derived equivalence between the orbit categories of dg categories with a group action, which will be illustrated in some examples. This is joint work with Asashiba Hideto.

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2020 Mathematics Subject Classification: 18G35, 16E35, 16E45, 16W22, 16W50

Keywords: Grothendieck construction, 2-category, colax functor, pseudofunctor, derived equivalence, dg category

Some aspect on various generalizations of $h-$Lifting Modules

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In this talk, we introduce and study the properties of *hw*−lifting modules as a special extensions of *h*−lifting which are further generalization of lifting modules. We observed that finite direct sum of *hw*−lifting modules may not be *hw*−lifting, so we provided several sufficient conditions for which *hw*−lifting modules are inherited under direct sum. Moreover, we have introduced and studied the properties of other versions of *h*−lifting module namely; *mh*−lifting, *mhw*−lifting and completely *mh*−lifting modules and established some more properties of the *hw*−lifting modules in terms of these modules. This is joint work with Laba Kr Das and K P Shum.

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2020 Mathematics Subject Classification: 16D10, 16D60, 16D70, 16D99

Keywords: Lifting module, hollow module, coessential submodule, maximal semisimple submodule

On the set of all generalized Drazin invertible elements in a ring

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Berkani and Sarihr [1] showed that the set of all Drazin invertible elements in an algebra over a filed is a regularity in the sense of Kordula and Muller [5]. In this note, the above result is extended to the case of a ring. Counterexamples are provided to show that the set of all generalized Drazin invertible elements in a ring need not be a regularity in general. We determine when the set of all generalized Drazin invertible matrices in the 2×2 full matrix ring over a commutative local ring is a regularity. We also give a sufficient condition for the set of all generalized Drazin invertible elements in a ring to be a regularity.

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2020 Mathematics Subject Classification: 16U90

Keywords: Regularity, Drazin inverse, generalized Drazin inverse, ring

Polynomial Dedekind Domains and Stacked Pseudo-convergent Sequences

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A Polynomial Dedekind Domain over the ring of integers $\mathbb{Z}$ is a Dedekind domain R contained between the polynomial rings $\mathbb{Z}[X]$ and $\mathbb{Q}[X]$. We provide a full characterization of such Dedekind domains, showing that they can be represented by means of rings of integer-valued polynomials. More precisely, for a prime $p \in \mathbb{Z}$, let $\mathbb{C}_p$ be the completion of the algebraic closure $\overline{\mathbb{Q}_p}$ of the field of p-adic numbers and let v_p denote the unique valuation on $\mathbb{C}_p$ extending the classical p-adic valuation. Given a Polynomial Dedekind Domain R over $\mathbb{Z}$, we show that for each prime $p \in \mathbb{Z}$ there exists a finite subset E_p of $\mathbb{C}_p$, whose elements are transcendental over $\mathbb{Q}$, such that the polynomials in R are precisely those which are simultaneously integer-valued over E_p for each prime p, that is, $R = \{f \in \mathbb{Q}[X] \mid v_p(f(E_p)) \geq 0, \forall \text{ prime } p\}$. We show that for each group G which is a direct sum of a countable family of finitely generated abelian groups, there exists a Polynomial Dedekind domain R over $\mathbb{Z}$ with class group G. In particular, we also obtain a characterization of the PIDs between $\mathbb{Z}[X]$ and $\mathbb{Q}[X]$.

This result is obtained by a characterization of residually algebraic torsion extensions of $\mathbb{Z}_{(p)}$ to $\mathbb{Q}(X)$ by means of a suitable kind of pseudo-convergent sequence in $\overline{\mathbb{Q}_p}$ called stacked. In particular, if W is a DVR of $\mathbb{Q}(X)$ extending $\mathbb{Z}_{(p)}$ such that the residue field extension is algebraic, there exists $\alpha \in \mathbb{C}_p$, transcendental over $\mathbb{Q}$, such that $W = \{\varphi \in \mathbb{Q}(X) \mid v_p(\varphi(\alpha)) \geq 0\}$. The residue field extension of W over $\mathbb{Z}_{(p)}$ is finite if and only if $\alpha \in \overline{\mathbb{Q}_p}$.

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2020 Mathematics Subject Classification: 12J20, 13F30, 13A18, 13F05, 13B25, 13F20

Keywords: Dedekind domain, integer-valued polynomial, pseudo-convergent sequence, residually algebraic extension

On class group of upper cluster algebras

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Cluster algebras are a class of commutative rings introduced by Fomin and Zelevinsky (2002) endowed with a family of distinguished generators, which are constructed recursively using mutations. Cluster algebras have been the focus of intense research since, thanks to the many links that have been discovered with a wide range of subjects, although their ring-theoretic properties are not so well-explored.

In this talk we focus on factorization-theoretic properties of upper cluster algebras, an upper bound for cluster algebras given by the Laurent phenomenon. We show that upper cluster algebras are completely integrally closed domains and FF-domains. Moreover we give a description of the class group of full rank upper cluster algebras in term of the exchange polynomials. This extends results of Garcia-Elsener–Lampe–Smertnig (2019) on acyclic cluster algebras and of Cao–Keller–Qin (2022) on full rank upper cluster algebras. This is a joint work with D. Smertnig.

2020 Mathematics Subject Classification: 13F60, 13F05, 13F15

Keywords: Cluster algebras, Krull domains, Class groups

From Derivation on Module Theory to Co-derivation on Comodule Theory

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Let (C, Δ, ε) be an R-algebra and M a left C-comodule. This research motivated by derivations on ring and module theory. We introduce co-derivation concept on coalgebra and comodule. By using the dualization concept from multiplication to comultiplication and action to coaction. We define some axioms for a linear map of coalgebra and comodule to become a co-derivation. We also see that a derivation of rings can be considering as a trivial co-derivation of coalgebra and a derivation of modules is a trivial co-derivation of comodule. However, by some steps and using an example we can prove that a ring derivation is not need to be a coalgebra co-derivation. We conclude that the comultiplication of coalgebras and coaction of comodules are important to ensure that a linear map from C to C and M to M be a co-derivation. This is joint work with Fitriani and Indah Emilia Wijayanti.

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2020 Mathematics Subject Classification: 16T15, 16D10

Keywords: derivation, co-derivation, modules, comodules, coalgebra

On the arithmetic of orders in algebraic number fields

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We give an overview of the arithmetic of orders in algebraic number fields. Let K be a number field with ring of integers $\mathcal{O}_K$ and let $\mathcal{O}$ be an order in K. From a factorization-theoretical viewpoint, $\mathcal{O}$ behaves similarly to the Dedekind domain $\mathcal{O}_K$, where the arithmetic is well-understood via the ideal class group. The main differences are due to a finite number of irregular prime ideals of $\mathcal{O}$, namely the prime ideals that contain the conductor of $\mathcal{O}$.

We call $\mathcal{O}$ half-factorial if every factorization into irreducibles of a given element has the same number of factors. We will give an algebraic characterization of half-factorial orders, which depends on the localizations $\mathcal{O}_{\mathfrak{p}}$ for the irregular pirme ideals $\mathfrak{p}$.

2020 Mathematics Subject Classification: 11R27, 13A05, 13F05

Keywords: orders, algebraic number fields, sets of lengths, half-factoriality

Factorization into valuation ideals and valuation elements

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Let D be an integral domain, let I be an ideal of D and let x be a nonzero nonunit of D. We say that I is a valuation ideal of D if there exists a valuation overring V of D such that $IV \cap D = I$. Moreover, x is said to be a valuation element of D if xD is a valuation ideal. The domain D is called a (weak) VIFD if every nonzero proper (principal) ideal of D is a finite product of valuation ideals of D. D is said to be a VFD if every nonzero nonunit of D is a finite product of valuation elements of D. Finally, we say that D is an almost VFD if for each nonzero nonunit $y \in D$, there is a positive integer n such that y^n is a finite product of valuation elements of D.

In this talk, we discuss the connections between (weak) VIFDs and (almost) VFDs. Based on earlier work (1.) on VFDs, we establish various similar results. We provide a characterization of treed (weak) VIFDs and mention several necessary conditions for D to be an almost VFD. Besides that, we present descriptions of almost VFDs and weak VIFDs in terms of almost Schreier domains and quasi Schreier domains, respectively. As an application, we characterize when the polynomial ring over D is an almost VFD. To supplement our results, we touch upon the t- and w-analogues of (weak) VIFDs. This is joint work with Gyu Whan Chang.

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2020 Mathematics Subject Classification: 13A15, 13F05, 13G05

Keywords: Almost Schreier domain, Prüfer domain, treed domain, valuation ideal

On the rudimentary property of rings

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In 2014, together with G. Lee and X. Zhang, we introduced the concept of rudimentary ring, a natural yet non-trivial generalization of that of the well-known primitive rings. A ring R is called right rudimentary if there exists a right R-module M such that M is faithful and $End_R(M)$ is a division ring. This new concept inherits some important properties from primitive rings, such as Morita invariance. Primitive rings are relatively easy to describe, by now a textbook exercise, as dense subrings in matrix rings or, in general, of endomorphism rings of vector spaces over division rings. However, due to their dependence on endomorphism rings of modules, rudimentary rings have a more complicated structure, not yet fully developed. In this talk I will provide new insights in the structure of rudimentary rings, indicate some applications, and give a number of examples that delimit these results and the included ancillary notions. This is a joint work with Gangyong Lee and Xiaoxiang Zhang.

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2020 Mathematics Subject Classification: 16K99, 16S50

Keywords: primitive ring, division ring, faithful module, endomorphism ring

Classifying Serre subcategories of the category of Maximal Cohen-Macaulay modules

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Classifying several subcategories is an active subject in the representation theory of algebras. One of the most classical results is Gabriel's classification of Serre subcategories. He classified Serre subcategories of the module category of a commutative noetherian ring in terms of its prime spectrum. So far, the classification of subcategories has focused on the case of abelian categories (or triangulated categories) such as the above example.

In this talk, we will talk about a classification of Serre subcategories of a certain exact category which is not abelian. Here, an exact category is a generalization of an abelian category, and this framework enables us to study homological properties of, for instance, the category of maximal Cohen-Macaulay modules, which is not an abelian category. We will give the classification result of Serre subcategories of a torsionfree class of the module category of a commutative noetherian ring. Here, a torsionfree class is a subcategory of an abelian category closed under subobjects and extensions, and it has a natural structure of an exact category. This result extends Gabriel's classification to torsionfree classes. As an immediate consequence, we can classify the Serre subcategories of the category of maximal Cohen-Macaulay modules over a one-dimensional Cohen-Macaulay ring.

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2020 Mathematics Subject Classification: 13C60, 16G50

Keywords: Serre subcategories, torsionfree classes, exact categories, Cohen-Macaulay modules

IE-closed subcategories

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In representation theory of algebras, one of the main topics is to study subcategories of a module category. Torsion classes and wide subcategories are particularly important and studied deeply. In this talk, we introduce IE-closed subcategories which unifies torsion classes, wide subcategories and so on. In the formar part of the talk, we characterize tau-tilting finiteness using IE-closed subcategories. In the latter part, we give a classification of IE-closed subcategories over a hereditary algebra using twin rigid modules, pairs of rigid modules satisfying some homological conditions.

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2020 Mathematics Subject Classification: 16G10, 18E40

Keywords: IE-closed subcategory, torsion (-free) class, tau-tilting finite algebra

On Gorenstein global and Gorenstein weak global dimensions

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It is well known that the weak global dimension of a ring does not exceed its global dimension. Christensen et al obtained a corresponding result in Gorenstein setting for coherent rings. We extend this result to an arbitrary ring. As applications, we characterize the finiteness of (Gorenstein) global dimension by singularity categories and Gorenstein flat modules. This is a joint work with Junpeng Wang, Gang Yang and Xiaoxiang Zhang.

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2020 Mathematics Subject Classification: 18G25, 16E65, 18G20

Keywords: Gorenstein (weak) global dimension, Gorenstein projective module, (projectively coresolving) Gorenstein flat module, singularity category

Inductive algebras for compact groups

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Inductive algebras for a compact group are self-adjoint. This is joint work with M. K. Vemuri.

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2020 Mathematics Subject Classification: 20C15

Keywords: Compact group, Representation, Inductive algebra, Schur's lemma

On noncommutative bounded factorization domains and prime rings

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A domain R is a bounded factorization (BF) domain if for every $0 \neq a \in R$, there exists $\lambda(a) \in \mathbb{N}_0$ such every factorization of a into atoms has length at most $\lambda(a)$.

It is well-known that commutative noetherian domains (and more generally, Mori domains) are BF-domains. The situation for noncommutative noetherian domains and prime rings is less clear, we present several recent sufficient conditions. This is joint work with Jason P. Bell, Ken Brown and Zahra Nazemian.

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[1] https://doi.org/10.1016/j.jalgebra.2023.01.023.

2020 Mathematics Subject Classification: Primary 16P40, Secondary 13F15, 16E65, 20M13

Keywords: bounded factorizations, BFD, noetherian prime rings, non-unique factorizations

Morita equivalences and relative stable equivalences of Morita type for the principal blocks of finite groups

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Let k be a field of characteristic $p > 0$, and G a finite group. Then the group algebra kG has a unique decomposition into a direct sum of indecomposable algebras. Each summand is called a block of kG. In particular, kG has a unique block that is called the principal block of kG.

Morita equivalences between principal blocks have been constructed by lifting stable equivalences of Morita type. The notion of a stable equivalence of Morita type was introduced by Broué [1]. He also developed a method of constructing one between principal blocks. Linckelmann [3] gave an equivalent condition for stable equivalences of Morita type between indecomposable selfinjective algebras to be in fact Morita equivalences. By combining these results, Morita equivalences have been constructed in some cases (see for example [4] and [2]). However if finite groups have a common nontrivial central p-subgroup, then we cannot use Broué's method for the principal blocks. In order to have a method of constructing Morita equivalences for principal blocks in the situation, we consider the notion of relative stable equivalences of Morita type.

The notion of relative stable equivalences of Morita type for blocks was introduced by Wang and Zhang [5]. This is a generalization of stable equivalences of Morita type. In this talk, we consider finite groups having a common central p-subgroup Z, and generalize the results due to Broué and Linckelmann to stable equivalences of Morita type relative to Z between the principal blocks. This is joint work with Naoko Kunugi.

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2020 Mathematics Subject Classification: 20C20

Keywords: Morita equivalence, Relative stable equivalence of Morita type

Classifying t-structures of derived categories

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In this talk we consider classifying t-structures of the bounded derived category $\mathrm{D}^{\mathrm{b}}(\operatorname{mod} R)$ of finitely generated modules over a commutative noetherian ring R. Alonso Tarrío, Jeremías López and Saorín give a complete classification of the t-structures of $\mathrm{D}^{\mathrm{b}}(\operatorname{mod} R)$ in terms of certain filtrations by supports of $\operatorname{Spec} R$ in the case where R admits a dualizing complex. We try to do the same classification in a more general setting. Our key tool is Faltings' annihilator theorem on local cohomology.

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2020 Mathematics Subject Classification: 13D09, 13D45, 13F40

Keywords: derived category, local cohomology, t-structure

Periodic dimensions for modules and algebras

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Let R be a Noetherian semiperfect ring. Eventually periodic R-modules are finitely generated R-modules having periodic higher syzygies. Until now, eventually periodic R-modules have been characterized in various ways in the literature such as [1, 2, 3, 4, 5, 6]. Simultaneously, the degrees and the periods of their first periodic syzygies have been also studied. See also [7] other than the above references.

The aim of this talk is to investigate the degree of the first periodic syzygy of an eventually periodic module. To do this, we will introduce the notion of the periodic dimension of a module, which indicates the desired degree when the module is eventually periodic. In this talk, after providing basic properties of periodic dimensions, we will give our main result that periodic dimensions can be computed from Gorenstein dimensions, and vice versa. This improves some known results on the degrees of the first periodic syzygies of eventually periodic modules over Gorenstien rings.

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2020 Mathematics Subject Classification: 16E05, 16E10, 16G10, 16G50

Keywords: eventually periodic modules, Gorenstein dimensions, eventually periodic algebras, Gorenstein algebras

On Differential Colocalizations on Coalgebras

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Differential torsion theories have already well known. For a cohereditary torsion theory τ, every derivation on an R-module can be lifted uniquely to its module of coquotients. Some concrete examples of cohereditary torsion theory in category of M-subgenerated modules $\sigma[M]$ has been given. In this work we will apply it to coalgebras.

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2020 Mathematics Subject Classification: 13C60

Keywords: Differential torsion theory, cohereditary torsion theory, module of coquotient, coalgebras

A new class of generalized inverses in rings with involution

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Let R be a $*$-ring and let a, w, $v \in R$. This paper contributes to define two new classes of generalized inverses, called the w-core inverse and the dual v-core inverse in R. An element $a \in R$ is w-core invertible if there exists some $x \in R$ such that $awx^2 = x$, $xawa = a$ and $(awx)^* = awx$. Such an x is called a w-core inverse of a, which encompasses several known generalized inverses such as the core inverse, the e-core inverse and the Moore-Penrose inverse. Several characterizations and representations for elements to be w-core invertible are given, based on $\{1, 3\}$-inverses, inverses along an element, (b, c)-inverses, projections, ideals and units. Also, the connections between the w-core inverse and other generalized inverses are given. This is joint work with Jianlong Chen and Huihui Zhu.

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2020 Mathematics Subject Classification: 16U90, 15A09

Keywords: Core inverses, Moore-Penrose inverses, the inverse along an element, semigroups with involution

The minimal weak Drazin inverses

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Let R be a ring. In 1978, Campbell and Meyer proposed the minimal rank weak Drazin inverse of a matrix. We extend it to the ring R and define the minimal weak Drazin inverse of an element in the ring. Furthermore, it is proved that the minimal weak Drazin inverse of an element in a ring exist if and only if it is Drazin invertible. Then, the set of minimal weak Drazin inverses of an element in a ring is described. Finally, some properties and characterizations of minimal weak Drazin inverses are given. This is joint work with Jianlong Chen.

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Keywords: Drazin inverse, minimal weak Drazin inverse, strongly-π regular element

Tilting theory for finite dimensional 1-Iwanaga-Gorenstein algebras

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A finite dimensional graded algebra $A = \bigoplus_{i=0}^{\ell} A_i$ is called graded d-Iwanaga-Gorenstein if the graded injective dimensions of A_A and ${}_AA$ are at most d. A finitely generated graded A-module M is called Cohen-Macaulay if it satisfies $\mathrm{Ext}^i_A(M, A) = 0$ for all $i > 0$. We denote by $\mathrm{CM}^{\mathbb{Z}}A$ the category of graded Cohen-Macaulay A-modules. Since this category is a Frobenius category, its stable category $\underline{\mathrm{CM}}^{\mathbb{Z}}A$ has a structure of a triangulated category.

In representation theory of graded Iwanaga-Gorenstein algebras, one of main concerns is to study the existence of tilting objects in $\underline{CM}^{\mathbb{Z}}A$. By tilting theory for algebraic triangulated categories [2], if $\underline{CM}^{\mathbb{Z}}A$ has a tilting object T, then there is an equivalence $\underline{CM}^{\mathbb{Z}}A \cong \mathrm{K}^{\mathrm{b}}(\mathrm{proj}\,\Gamma)$ of triangulated categories, where $\Gamma = \mathrm{End}_{\underline{CM}^{\mathbb{Z}}A}(T)$ and $\mathrm{K}^{\mathrm{b}}(\mathrm{proj}\,\Gamma)$ is the homotopy category of bounded complexes of projective Γ-modules. This equivalence allows us to study Cohen-Macaulay A-modules from viewpoint of representations of Γ. Therefore, a fundamental problem is that when $\underline{CM}^{\mathbb{Z}}A$ has a tilting object.

In the case $d = 0$, the problem was solved. It was shown that $\underline{CM}^{\mathbb{Z}}A$ has a tilting object if and only if the 0-th subring A_0 of A is of finite global dimension. In the case $d \geq 1$, many authors studied the problem for various classes of (not necessarily finite dimensional) graded Iwanaga-Gorenstein algebras (refer to a survey article [1]).

In this talk, we develop tilting theory for finite dimensional 1-Iwanaga-Gorenstein algebras. We will introduce an invariant $g(A)$ for a finite dimensional graded algebra A. Then, we show that in the case where A is 1-Iwanaga-Gorenstein, an inequality for $g(A)$ gives a sufficient condition that a specific Cohen-Macaulay module V becomes a tilting object in $\underline{\mathrm{CM}}^{\mathbb{Z}}A$.

As an application, we study the problem for the case that A is the truncated preprojective algebra $\Pi(Q)_w$ of a quiver Q associated to $w \in W_Q$. We show that if the underling graph of Q is tree, then $\underline{\mathrm{CM}}^{\mathbb{Z}}\Pi(Q)_w$ has a tilting object. This is joint work with Yuta Kimura and Hiroyuki Minamoto.

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2020 Mathematics Subject Classification: 16G10, 16B50, 16E35, 16E65

Keywords: Iwanaga-Gorenstein algebra, Cohen-Macaulay module, tilting theory, derived category, stable category, singularity category

Some Properties of Left r-Clean Bimodules

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Let R be an associative ring with identity and M an R-bimodule. Ring R is considered clean if each element is the sum of idempotent and unit elements. By generalizing the unit elements to regular elements, the definition of a clean ring has been generalized to an r-clean ring. Previous researchers have developed a clean ring definition and their generalization to the module structure through its endomorphism ring. This research introduces the generalization of r-clean rings, called left r-clean R-bimodules, defined without their endomorphism rings. Furthermore, we present some of their properties and give the sufficient and necessary conditions for an R-bimodule to be r-clean. This is joint work with Indah Emilia Wijayanti and Budi Surodjo.

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Keywords: r-clean ring, regular element, R-bimodule, idempotent element

On S-1-absorbing prime and weakly S-1-absorbing prime ideals

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Let A be a commutative ring with identity and S be a multiplicative subset of A. In this talk, we introduce and study the notions of S-1-absorbing and weakly S-1-absorbing prime ideals as a generalizations of the notion of prime ideals. We define a proper ideal I disjoint with S to be an S-1-absorbing (resp. a weakly S-1-absorbing) prime ideal if there exists $s \in S$ such that for all nonunit elements $a, b, c \in A$ such that $abc \in I$, we have $sab \in I$ or $sc \in I$. Several properties and characterizations of S-1-absorbing prime and weakly S-1-basorbing prime ideals are given. Moreover, we study the transfer of the above properties to some constructions rings such as trivial ring extensions and amalgamation of rings along an ideal.

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2020 Mathematics Subject Classification: 13A15

Keywords: S-1-absorbing prime ideal, weakly S-1-absorbing prime ideal, S-prime, trivial extension

On the uniformity in rings, modules and homological algebras

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Let R be a ring with identity and S a multiplicative subset of R. We introduce the uniformly S-torsion theory, and utilize it to generalize various of classical notions appeared in rings, modules and homological algebras.

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2020 Mathematics Subject Classification: 13C12, 16S90, 16E30, 16D40, 13C11

Keywords: u-S-torsion module, u-S-exact sequence, u-S-artinian module, u-S-noetherian module, u-S-coherent module, u-S-homological dimensions

Birkhoff-Pierce Problem and Riemann Hypothesis

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In this talk, we introduce the brief history of the Birkhoff-Pierce Problem, so-called "No.1 problem of ordered algebraic structure", and give the details of the concept "doubly convex set" introduced by Zhipeng Xu, a 2nd year undergraduate student of SJTU. The unified classification of directed partial orders over F(i) (where F is any non-archimedean o-field and $i^2 = -1$) is obtained using this concept. We also explain a connection of Birkhoff-Pierce Problem and Riemann Hypothesis.

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2020 Mathematics Subject Classification: Primary 06F25, Secondary 20M25

Keywords: Birkhoff-Pierce Problem, Non-archimedean o-eld, Directed partially ordered algebra, Riemann Hypothesis, Doubly convex set

Some characterizations of w-Noetherian rings and SM rings

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In this paper, we characterize w-Noetherian rings and SM rings. More precisely, in terms of the u-operation on a commutative ring R, we prove that R is w-Noetherian if and only if the direct limit of rGV-torsion-free injective R-modules is injective and that R is SM, which can be regarded as a regular w-Noetherian ring, if and only if the direct limit of GV-torsion-free (or rGV-torsion-free) reg-injective R-modules is reg-injective. As a by-product of the proof of the second statement, we also obtain that the direct and inverse limits of u-modules are both u-modules and that SM rings are regular w-coherent. This is joint work with Hwankoo Kim, Xiao Lei Zhang and Jin Xie.

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2020 Mathematics Subject Classification: 13C99, 13A15

Keywords: *u*-operation, *w*-Noetherian ring, SM ring, regular ideal

Frobenius-Perron Theory of Finite Representations of Quivers

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The Frobenius-Perron theory of an abelian category with an endofunctor was introduced recently in *Frobenius-Perron Theory of Endofunctors*. We apply this theory to the finite representation categories and bounded derived categories of finite acyclic quivers. We show that if two derived categories are equivalent as tensor triangulated category, then these two quivers are isomorphic. At last, we calculate the Frobenius-Perron dimensions for $\mathbb{ADE}$ quivers as examples.

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Keywords: Frobenius–Perron dimension, Derived categories, Quiver representation, Monoidal triangulated category, ADE Dynkin quiver

Drazin inverses and pseudo core inverses of a sum of morphisms

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Let $\mathscr{C}$ be an additive category. Suppose that $\varphi : X \longrightarrow X$ is a morphism of $\mathscr{C}$ with Drazin inverse φ^D, and $\eta : X \longrightarrow X$ is a morphism of $\mathscr{C}$ such that $1+\varphi^D\eta$ is invertible and $\eta(\varphi^D\varphi - 1)\varphi = \varphi(\varphi^D\varphi - 1)\eta = 0$. Let $\alpha = (1+\varphi^D\eta)^{-1}$, $\beta = (1+\eta\varphi^D)^{-1}$, $\varepsilon = (1-\varphi\varphi^D)\eta\alpha(1-\varphi^D\varphi)$, $\gamma = \alpha(1-\varphi^D\varphi)\eta\varphi^D\beta$, $\sigma = \alpha\varphi^D\varphi\alpha^{-1}(1-\varphi\varphi^D)\beta$.It is proved that $f = \varphi + \eta - \varepsilon$ is Drazin invertible with $f - f^2 f^D = \varphi - \varphi^2\varphi^D$ if and only if $1-\gamma$ is invertible if and only if $1-\sigma$ is invertible. This result extends the case of group inverses and reduces the two invertible morphisms used by Chen et al. to one. The relevant result for pseudo core inverses of a sum of morphisms is also given. The talk reports a joint work with Jianlong Chen.

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Keywords: Drazin inverse, pseudo core inverse, additive category, morphism


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