QUATERNION RING AND APPLICATION IN HYPERNORMAL FORM OF 4 DIMENSIONAL SEMI-SIMPLE NONLINEAR DYNAMICAL SYSTEMS

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Quaternion in the real number domain was first proposed in 1843 by Hamilton, whose purpose was to find a way to study spatial geometry similar to complex number in solving plane problems [1]. As a famous example, quaternion plays an important role in the ring theory. Many experts and scholars have studied the theory and related properties of quaternion ring [2, 3]. Due to its unique properties and advantages [4], quaternion ring theory has potential application prospects in many fields [5].

In the field of nonlinear dynamics, further reduction of normal form or hypernormal form (unique normal form, simplest normal form) has become one of the most important topics [6, 7]. However, there are still very few results for further reduction of normal forms for higher dimensional systems. One of the main difficulties is that the matrices in the computation of normal forms are usually very large and which makes the computation very difficult. In this paper, we present a new method of expressing and simplifying high dimensional nonlinear dynamical systems by introducing the quaternion ring theory, and investigate the hypernormal form of a 4 dimensional semi-simple nonlinear dynamical system. The main technique used to the computation is the combination of a new grading function and multiple Lie brackets. The introduction of quaternion ring theory helps to reduce the computation of large size matrices in the study of hypernormal forms.

The research project is supported by National Natural Science Foundation of China (11772007, 11372014, 11802200) and also supported by Beijing Natural Science Foundation (1172002, Z180005).

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2010 *Mathematics Subject Classification.* 70K45, 30G05, 15B33.