On liftable DG modules over a commutative DG algebra

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This talk is based on a join work with Yuji Yoshino [4].

M.Auslander, S.Ding and Ø.Solberg [1] studied liftings and weak liftings of finitely generated modules over a commutative Noetherian algebra. Recently, S.Nasseh and S.Sather-Wagstaff [2] and S.Nasseh and Y.Yoshino [3] extended them to the case of differential graded (=DG) modules over differential graded (=DG) algebras.

Let $A \to B$ be a homomorphism of DG algebras over a commutative ring R. A DG B-module N is said to be *liftable to A* if there is a DG A-module M such that $N \cong B \otimes_A M$. In this case, M is called a *lifting of N* to A.

We concern a lifting problem in the situation $A \to B$ where $B = A\langle X | dX = t \rangle$ is an extended DG R-algebra of A by the adjunction of a variable X which kills a cycle t in A. The both papers [2, 3] treated the lifting problem or the weak lifting problem in such cases but with the assumption that the degree of X is odd. In this case, B is a Koszul complex over A.

In this talk, we study the lifting problem in the situation $A \to B = A\langle X | dX = t \rangle$ where B is obtained from adding a variable X of *even* degree. It should be noted that underlying graded algebra of B is a free algebra over A with a divided powers variable X. Let N be a semi-free DG B-module. The aim of this talk is to construct an obstruction for liftings of N to A as an element of $\mathrm{Ext}_B^{|X|+1}(N, N)$. To do this, we introduce a certain operator on the set of graded R-linear endomorphisms $\mathrm{End}_R(N)$, which is called the *j-operator*. The notion of the j-operator was first introduced by J.Tate in the paper [5]. We show that N is liftable to A if and only if the obstruction of N vanishes. The following is our main result of this talk.

Theorem 1. *Let A be a DG R-algebra and $B = A\langle X | dX = t \rangle$ be an extended DG R-algebra of A by the adjunction of a variable X of even positive degree. Assume that a DG B-module N is semi-free.*

(1) *If N is bounded below and $\mathrm{Ext}_B^{|X|+1}(N, N) = 0$, then N is liftable to A.*

(2) *If N is liftable to A and $\mathrm{Ext}_B^{|X|}(N, N) = 0$, then a lifting of N is unique up to DG A-isomorphisms.*

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