On CRP rings

Tai Keun Kwak
Daejin University
Email: tkkwak@daejin.ac.kr

We study the one-sided regularity of matrices in upper triangular matrix rings in relation with the structure of diagonal entries. We consider next a ring theoretic condition that ab being regular implies ba being also regular for elements a, b in a given ring. Rings with such a condition are said to be *commutative at regular product* (simply, *CRP* rings). CRP rings are shown to be contained in the class of directly finite rings, and we prove that if R is a directly finite ring that satisfies the descending chain condition for principal right ideals or principal left ideals, then R is CRP. We obtain in particular that the upper triangular matrix rings over commutative rings are CRP.

References

1. D.D. Anderson, V. Camillo, *Semigroups and rings whose zero products commute*, Comm. Algebra **27** (1999), 2847-2852.
2. R. Antoine, *Nilpotent elements and Armendariz rings*, J. Algebra **319** (2008), 3128–3140.
3. R. Baer, *Inverses and zero-divisors*, Bull. Amer. Math. Soc. **48** (1942), 630–638.
4. H.E. Bell, *Near-rings in which each element is a power of itself*, Bull. Austral. Math. Soc. **2** (1970), 363–368.
5. C. Huh, H.K. Kim, Y. Lee, *p.p. rings and generalized p.p. rings*, J. Pure Appl. Algebra **167** (2002), 37–52.
6. C. Huh, N.K. Kim, Y. Lee, *Examples of strongly π-regular rings*, J. Pure Appl. Algebra **189** (2004), 195–210.
7. C. Huh, Y. Lee, A. Smoktunowicz, *Armendariz rings and semicommutative rings*, Comm. Algebra **30** (2002), 751–761.
8. S.U. Hwang, N.K. Kim, Y. Lee, *On rings whose right annihilators are bounded*, Glasgow Math. J. **51** (2009), 539–559.
9. N. Jacobson, *Some remarks on one-sided inverses*, Proc. Amer. Math. Soc. **1** (1950), 352–355.
10. Y.C. Jeon, H.K. Kim, Y. Lee, J.S. Yoon, *On weak Armendariz rings*, Bull. Korean Math. Soc. **46** (2009), 135–146.
11. D.W. Jung, N.K. Kim, Y. Lee, S.P. Yang, *Nil-Armendariz rings and upper nilradicals*, Int. J. Alg. Comp. **22** (2012), 1–13(1250059).
12. J.C. Shepherdson, *Inverses and zero-divisors in matrix ring*, Proc. London Math. Soc. **3** (1951), 71–85.i

---

2010 *Mathematics Subject Classification.* 16U80, 16S50