On gendo-symmetric algebras of a trivial extension algebra.

Takahiro Honma. j.w. T. Aihara and A. Chan.

Motivation

Investigate rep-finiteness of $\mathrm{End}_A(A \oplus M)$

$K = \overline{K}$: field
A: f.d. K-alg
$D(-) := \mathrm{Hom}_K(-, K)$
$\mathrm{mod}\,A$: The cat of f.g. right A-modules
$T(A) := A \ltimes DA$: The trivial extension alg of A.

Def

Gendo-symmetric alg Λ over A is $\mathrm{End}_A(A \oplus M)$ where A: symm alg, M: A-module.

Q. When is a gendo-sym alg rep-fin?

Rem Gendo-symm Λ over A is rep-fin $\Rightarrow$ symm alg A is rep-fin.

☆ Rep-fin sym alg is the following cases.

- Brauer tree alg
- Modified Brauer tree alg
- Rep-fin triv ext alg ← In this talk.

Prop [Happel, Riedtmann]

- $T(A)$ is rep-fin $\Leftrightarrow$ $\exists$ Dynkin quiver $\vec{\Delta}$ s.t. $A \overset{\text{der}}{\sim} K\vec{\Delta}$

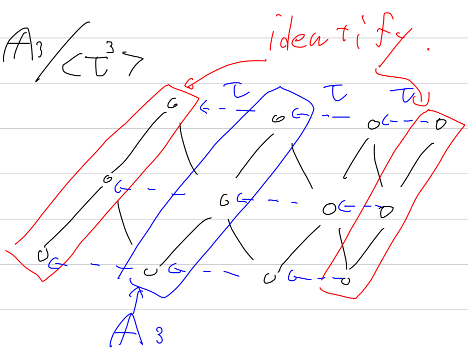
In the case,

$${}_s\Gamma(\mathrm{mod}\,T(A)) \simeq \mathbb{Z}\Delta/\langle \tau^{h(\Delta)-1} \rangle$$

(stable AR-quiver of $T(A)$; $h(\Delta)$: coxceter number)

Ex. $\Delta = A_3 = \circ \longrightarrow \circ \longrightarrow \circ$

$\mathbb{Z}A_3/\langle \tau^3 \rangle$



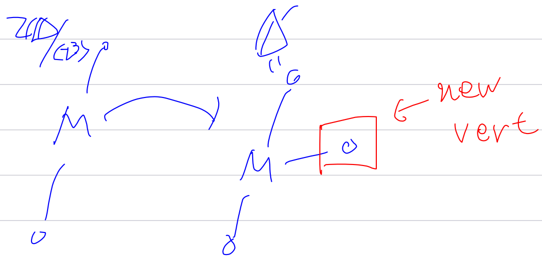
Thm [Aihara-Chan-H]

A: alg s.t. $A \overset{\text{der}}{\sim} K\vec{\Delta}$ where $\vec{\Delta}$ is Dynkin quiver.

M: ind non-proj $T(A)$-module.

$\Lambda := \mathrm{End}_{T(A)}(T(A) \oplus M)$: Gendo alg

Define Δ' as the diagram given by extending Δ at the vertex corresp to M in $\mathbb{Z}\Delta/\langle\ \rangle$)



$\exists$ alg B s.t.
- $B \overset{\text{der}}{\sim} K\vec{\Delta'}$
- $\Lambda \cong T(B)/\mathrm{soc}\,P$ where P: ind proj $T(B)$-module. corresp to the new vertex

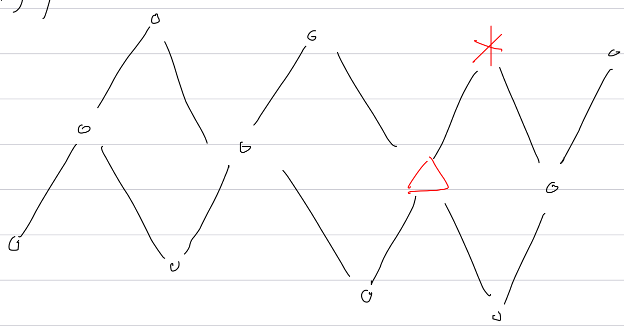
In particular Λ is rep-fin $\Leftrightarrow$ Δ' is Dynkin diagram

rep-tame $\Leftrightarrow$ Δ' is Euclidean.

Example

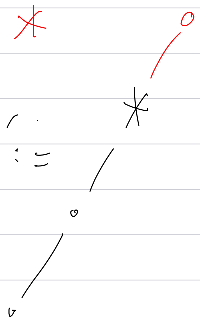
- $A \overset{\text{der}}{\sim} KA_3$

${}_s\Gamma(\mathrm{mod}\,T(A))$



- $M = *$

$\Delta' :=$

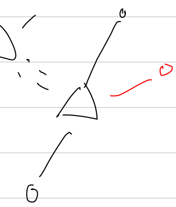


$\leftarrow A_4$

$\therefore \Lambda$ is rep-fin

- $M = \Delta$

$\Delta' :=$

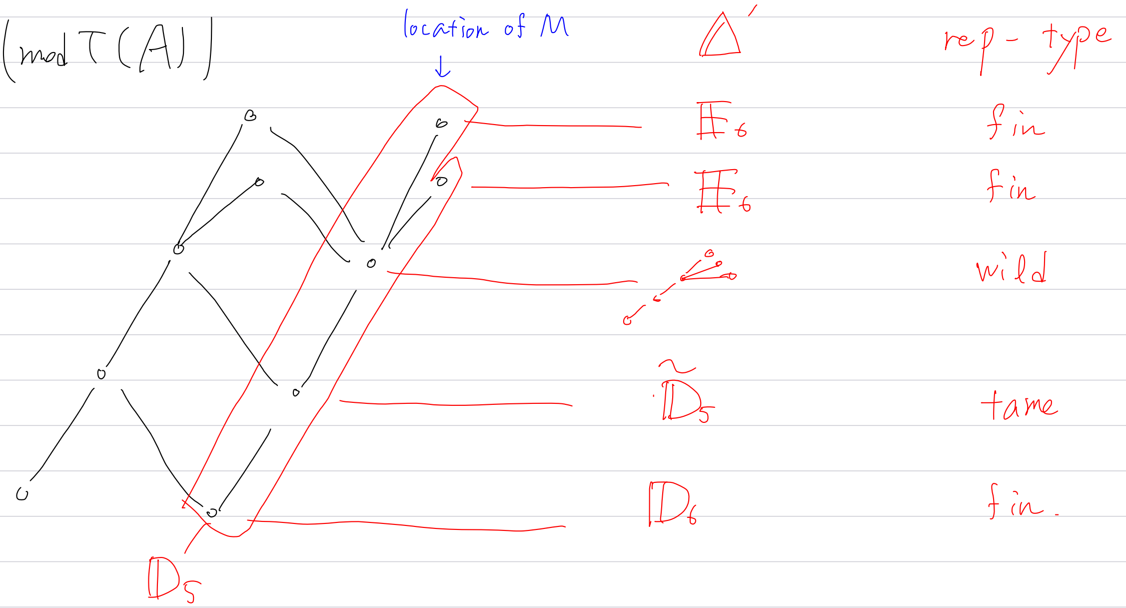


$\leftarrow D_4$

$\therefore \Lambda$ is rep-fin

• Let $A \overset{der}{\simeq} K\overrightarrow{\mathbb{D}_5}$.

${}_s\Gamma(\mathrm{mod}\,T(A))$



Rem

① [Boehlmer]: Classification of rep-fin gendo-symm $\mathrm{End}(A \oplus M)$ with M: indec
A: Brauer Tree with non-trivial multiplicity.

② We also have the classification of rep-fin gendo-symm $\mathrm{End}_A(A \oplus M)$ with M: indec
and A: modified Br.