Lie solvability in matrix algebras

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If an algebra $\mathcal{A}$ satisfies the polynomial identity

$$[x_1, y_1][x_2, y_2] \cdots [x_{2^m}, y_{2^m}] = 0$$

(for short, $\mathcal{A}$ is D_{2^m}), then $\mathcal{A}$ is trivially Lie solvable of index $m+1$ (for short, $\mathcal{A}$ is Ls_{m+1}). We will show that the converse holds for subalgebras of the upper triangular matrix algebra $U_n(R)$, R any commutative ring, and $n \geq 1$.

We will also consider two related questions, namely whether, for a field F, an Ls_2 subalgebra of $M_n(F)$, for some n, with (F-)dimension larger than the maximum dimension $2 + \left\lfloor \frac{3n^2}{8} \right\rfloor$ of a D_2 subalgebra of $M_n(F)$, exists, and whether a D_2 subalgebra of $U_n(F)$ with (the mentioned) maximum dimension, other than the typical D_2 subalgebras of $U_n(F)$ with maximum dimension, which were exhibited in [1] and refined in [3], exists. Partial results with regard to these two questions are obtained.

References

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