# Happel's functor and homologically well-graded Iwanaga-Gorenstein algebras

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In representation theory of algebras, derived category and stable category are two major classes of triangulated categories. It has been shown by many researchers that those different kinds of triangulated categories are related in various cases.

Happel [2] established the following relationship. For a finite dimensional algebra $\Lambda$ over a field, one has a trivial extension $\mathrm{T}(\Lambda) = \Lambda \oplus D(\Lambda)$. $\mathrm{T}(\Lambda)$ is a graded self-injective algebra, and so the stable category $\underline{\mathrm{mod}}^{\mathbb{Z}}\,\mathrm{T}(\Lambda)$ of $\mathbb{Z}$-graded $\mathrm{T}(\Lambda)$-modules has a structure of triangulated category. In this setting, he constructed a fully faithful functor

$$\mathcal{H} : \mathsf{D}^{\mathrm{b}}(\mathrm{mod}\,\Lambda) \to \underline{\mathrm{mod}}^{\mathbb{Z}}\,\mathrm{T}(\Lambda).$$

He also showed that $\mathcal{H}$ gives an equivalence precisely when $\mathrm{gl.dim}\,\Lambda < \infty$.

This functor $\mathcal{H}$ can be generalized as follows. We start from a finitely graded Iwanaga-Gorenstein algebra $A$, and replace $\underline{\mathrm{mod}}^{\mathbb{Z}}\,\mathrm{T}(\Lambda)$ with the stable category $\underline{\mathsf{CM}}^{\mathbb{Z}}A$ of $\mathbb{Z}$-graded Cohen-Macaulay $A$-modules. Also we replace $\Lambda$ with the Beilinson algebra $\nabla A$ of $A$. Then there is a functor

$$\mathcal{H} : \mathsf{D}^{\mathrm{b}}(\mathrm{mod}\,\nabla A) \to \underline{\mathsf{CM}}^{\mathbb{Z}}\,A.$$

Note that this is not fully faithful in general.

In my talk, we study when this functor $\mathcal{H}$ is fully faithful or gives an equivalence. For this purpose, we introduce *homologically well-graded* Iwanaga-Gorenstein algebra, which can be characterized as a finitely graded algebra posses a homological symmetry. Our main result is that this class of algebras is precisely the class of finitely graded Iwanaga-Gorenstein algebras $A$ that $\mathcal{H}$ is fully faithful. We also identify the class that $\mathcal{H}$ gives an equivalence. Our results recover some of results shown in previous works [1, 2, 3].

## References

1. X. W. Chen, *Graded self-injective algebras "are" trivial extensions*, J. Algebra 322 (2009), 2601–2606.
2. D. Happel, *On the derived category of a finite-dimensional algebra*, Comment. Math. Helv. 62 (1987), no. 3, 339–389.
3. M. Lu, *Singularity categories of representations of quivers over local rings*, arXiv:1702.01367.
4. H. Minamoto, K. Yamaura, *Happel's functor and homologically well-graded Iwanaga-Gorenstein algebras*, arXiv:1811.08036.

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