ON A PROBLEM OF SOCLE-DEFORMATIONS OF
SELF-INJECTIVE ORBIT ALGEBRAS

Kunio Yamagata
Tokyo University of Agriculture and Technology

Email: yamagata@cc.tuat.ac.jp

This is a report from joint work with A. Skowroński, [1] [2].

By an algebra we mean a basic, connected, finite-dimensional associative algebra with
identity over a field $K$. For an algebra $A$, we consider finite-dimensional right $A$-modules,
and denote by mod $A$ the category of finite-dimensional right $A$-modules.

An algebra $A$ is called selfinjective if $A$ is injective in mod $A$, and then soc($A$) :=
soc(A_A) = soc(A_A).$ Selfinjective algebras $A$ and $A'$ are said to be socle equivalent if the
quotient algebras $A/soc(A)$ and $A'/soc(A')$ are isomorphic, in this case, $A$ is also called
a socle deformation of $A'$.

Let $\hat{B}$ be the repetitive algebra of an algebra $B$, which is an infinite dimensional $K$-
algebra with $\bigoplus_{i \in \mathbb{Z}} (B \oplus D(B))$ as a $K$-vector space, where $D(B) = \text{Hom}_K(B,K)$. For
some group $G$ of automorphisms of $\hat{B}$ regarded as a $K$-category, we have the category
$\hat{B}/G$ whose objects are by definition all $G$-orbits of objects of $\hat{B}$, and $\hat{B}/G$ as an
algebra is finite dimensional selfinjective, called an orbit algebra of $B$. Important classes
of socle deformations $A$ of a selfinjective orbit algebra $\hat{B}/G$ are of finite representation
type (C. Riedtmann, 1980-83) and of polynomial growth (A, Skowroński, 1989) over an
algebraically closed field $K$, in those cases $B$ may be chosen as an algebra of finite global
dimension and $G$ an infinite cyclic group. In fact, $B$ is a quasi-tilted algebra (more
precisely, a tilted algebra for $A$ of representation-finite or representation-domestic type
(special case of polynomial growth)).

Problem: Determine the selfinjective algebras $A$ over a filed $K$ socle equivalent to an
orbit algebra $\hat{B}/G$ of an algebra $B$ of finite global dimension and $G$ an infinite cyclic
group.

It should be noted that the problem asserts that the study of selfinjective algebras
$A$ determined in the problem may be reduced to the study of algebras $B$ of finite global
dimension, and it seems to be difficult even in the case of representation-finite selfinjective
algebras over a (not necessarily algebraically closed) field.

In my talk, in view of the above facts by Riedtmann and Skowróński, we consider
the case where $B$ is a tilted algebra and $G$ is a cyclic group generated by an automor-
phism of the form $\varphi \nu_B$ where $\nu_B$ and $\varphi$ are the Nakayama and a positive autormorphisms
respectively, and a solution to the case and applications are explained.

References
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2. A. Skowróński and K. Yamagata, Socle deformations of selfinjective orbit algebras of tilted type, arX-
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