

ON A PROBLEM OF SOCLE-DEFORMATIONS OF SELF-INJECTIVE ORBIT ALGEBRAS

Kunio Yamagata

Tokyo University of Agriculture and Technology

Email: yamagata@cc.tuat.ac.jp

This is a report from joint work with A. Skowroński, [1] [2].

By an algebra we mean a basic, connected, finite-dimensional associative algebra with identity over a field K . For an algebra A , we consider finite-dimensional right A -modules, and denote by $\text{mod } A$ the category of finite-dimensional right A -modules.

An algebra A is called *selfinjective* if A is injective in $\text{mod } A$, and then $\text{soc}(A) := \text{soc}(A_A) = \text{soc}({}_A A)$. Selfinjective algebras A and A' are said to be *socle equivalent* if the quotient algebras $A/\text{soc}(A)$ and $A'/\text{soc}(A')$ are isomorphic, in this case, A is also called a *socle deformation* of A' .

Let \widehat{B} be the repetitive algebra of an algebra B , which is an infinite dimensional K -algebra with $\bigoplus_{i \in \mathbb{Z}} (B \oplus D(B))$ as a K -vector space, where $D(B) = \text{Hom}_K(B, K)$. For some group G of automorphisms of \widehat{B} regarded as a K -category, we have the category \widehat{B}/G whose objects are by definition all G -orbits of objects of \widehat{B} , and \widehat{B}/G as an algebra is finite dimensional selfinjective, called an *orbit algebra* of B . Important classes of socle deformations A of a selfinjective orbit algebra \widehat{B}/G are of finite representation type (C. Riedtmann, 1980-83) and of polynomial growth (A. Skowroński, 1989) over an algebraically closed field K , in those cases B may be chosen as an algebra of finite global dimension and G an infinite cyclic group. In fact, B is a quasi-tilted algebra (more precisely, a tilted algebra for A of representation-finite or representation-domestic type (special case of polynomial growth)).

Problem: *Determine the selfinjective algebras A over a field K socle equivalent to an orbit algebra \widehat{B}/G of an algebra B of finite global dimension and G an infinite cyclic group.*

It should be noted that the problem asserts that the study of selfinjective algebras A determined in the problem may be reduced to the study of algebras B of finite global dimension, and it seems to be difficult even in the case of representation-finite selfinjective algebras over a (not necessarily algebraically closed) field.

In my talk, in view of the above facts by Riedtmann and Skowroński, we consider the case where B is a tilted algebra and G is a cyclic group generated by an automorphism of the form $\varphi\nu_{\widehat{B}}$ where $\nu_{\widehat{B}}$ and φ are the Nakayama and a positive automorphisms respectively, and a solution to the case and applications are explained.

REFERENCES

1. A. Skowroński and K. Yamagata, *Selfinjective algebras with hereditary stable slice*, J. Algebra **530** (2019), 146–168
2. A. Skowroński and K. Yamagata, *Socle deformations of selfinjective orbit algebras of tilted type*, arXiv:1905.034921.