A Batalin-Vilkovisky differential on the complete cohomology ring of a Frobenius algebra

Tomohiro Itagaki, Katsunori Sanada and Satoshi Usui

Tokyo University of Science

Email: titagaki@rs.tus.ac.jp, sanada@rs.tus.ac.jp, 1119702@ed.tus.ac.jp

In the 1980s, Buchweitz [1] introduced the notion of singularity category in order to provide a framework for Tate cohomology of Gorenstein algebras. Recently, under this framework, Wang [3] has defined the r-th *Tate-Hochschild cohomology group* of a Noetherian algebra A over a field k as

$$\underline{\mathrm{Ext}}^r_{A\otimes_k A^{\mathrm{op}}}(A, A) := \mathrm{Hom}_{\mathcal{D}_{\mathrm{sg}}(A\otimes_k A^{\mathrm{op}})}(A, A[r]),$$

where $r \in \mathbb{Z}$ and $\mathcal{D}_{\mathrm{sg}}(A \otimes_k A^{\mathrm{op}})$ is the singularity category of $A \otimes_k A^{\mathrm{op}}$. He also discovered a Gerstenhaber structure on the Tate-Hochschild cohomology ring

$$\underline{\mathrm{Ext}}^{\bullet}_{A\otimes_k A^{\mathrm{op}}}(A, A) := \bigoplus_{r\in\mathbb{Z}} \underline{\mathrm{Ext}}^r_{A\otimes_k A^{\mathrm{op}}}(A, A).$$

In 1957, Nakayama [2] introduced the complete cohomology groups $\widehat{\mathrm{HH}}^*(A, A)$ of a Frobenius algebra A over a field k, which is analogous to Tate cohomology of a finite group. It is known that the complete cohomology is isomorphic to the Tate-Hochschild cohomology. Wang [3] proved that there is a graded commutative product $\star$, called $\star$-product, on the complete cohomology such that the complete cohomology ring is isomorphic to Tate-Hochschild cohomology ring. Moreover, he showed that the complete cohomology ring of a symmetric algebra has a Batalin-Vilkovisky (BV) structure by using Tradler's BV differential and Connes operator. In particular, the BV differential generates the Gerstenhaber bracket on the Tate-Hochschild cohomology.

In this talk, we explain how to construct a BV structure on the complete cohomology of a Frobenius algebra whose Nakayama automorphism is diagonalizable.

References

[1] R.-O. Buchweitz, *Maximal Cohen-Macaulay modules and Tate-cohomology over Gorenstein rings*, preprint, 1986, https://tspace.library.utoronto.ca/handle/1807/16682.

[2] T. Nakayama, *On the Complete Cohomology Theory of Frobenius Algebras*, Osaka Math. J. **9** (1957) 165–187.

[3] Z. Wang, *Gerstenhaber algebra and Deligne's conjecture on Tate-Hochschild cohomology*, https://arxiv.org/abs/1801.07990, 2018.

2010 *Mathematics Subject Classification.* 16E40, 16E45.