A Batalin-Vilkovisky differential on the complete cohomology ring of a Frobenius algebra

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In the 1980s, Buchweitz [1] introduced the notion of singularity category in order to provide a framework for Tate cohomology of Gorenstein algebras. Recently, under this framework, Wang [3] has defined the \( r \)-th Tate-Hochschild cohomology group of a Noetherian algebra \( A \) over a field \( k \) as

\[
\text{Ext}^r_{A \otimes_k A^{op}}(A, A) := \text{Hom}_{\mathcal{D}_{sg}(A \otimes_k A^{op})}(A, A[r]),
\]

where \( r \in \mathbb{Z} \) and \( \mathcal{D}_{sg}(A \otimes_k A^{op}) \) is the singularity category of \( A \otimes_k A^{op} \). He also discovered a Gerstenhaber structure on the Tate-Hochschild cohomology ring

\[
\text{Ext}^\bullet_{A \otimes_k A^{op}}(A, A) := \bigoplus_{r \in \mathbb{Z}} \text{Ext}^r_{A \otimes_k A^{op}}(A, A).
\]

In 1957, Nakayama [2] introduced the complete cohomology groups \( \widehat{\text{HH}}^\bullet(A, A) \) of a Frobenius algebra \( A \) over a field \( k \), which is analogous to Tate cohomology of a finite group. It is known that the complete cohomology is isomorphic to the Tate-Hochschild cohomology. Wang [3] proved that there is a graded commutative product \( \ast \), called \( \ast \)-product, on the complete cohomology such that the complete cohomology ring is isomorphic to Tate-Hochschild cohomology ring. Moreover, he showed that the complete cohomology ring of a symmetric algebra has a Batalin-Vilkovisky (BV) structure by using Tradler’s BV differential and Connes operator. In particular, the BV differential generates the Gerstenhaber bracket on the Tate-Hochschild cohomology.

In this talk, we explain how to construct a BV structure on the complete cohomology of a Frobenius algebra whose Nakayama automorphism is diagonalizable.

REFERENCES


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