Knörrer's periodicity for skew quadric hypersurfaces

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It is well-known that A is the homogeneous coordinate ring of a smooth quadric hypersurface in $\mathbb{P}^{n-1}$ if and only if $A \cong k[x_1, \dots, x_n]/(x_1^2 + \cdots + x_n^2)$. Applying the graded Knörrer's periodicity theorem, we have

$$\underline{\mathrm{CM}}^{\mathbb{Z}}(A) \cong \begin{cases} \underline{\mathrm{CM}}^{\mathbb{Z}}(k[x_1]/(x_1^2)) \cong \mathcal{D}^b(\operatorname{mod} k) & \text{if } n \text{ is odd,} \\ \underline{\mathrm{CM}}^{\mathbb{Z}}(k[x_1, x_2]/(x_1^2 + x_2^2)) \cong \mathcal{D}^b(\operatorname{mod} k^2) & \text{if } n \text{ is even.} \end{cases}$$

In this talk, we study a skew version of this equivalence.

Let $S = k\langle x_1, \dots, x_n\rangle/(x_i x_j - \varepsilon_{ij} x_j x_i)$ be a (± 1)-skew polynomial algebra generated in degree 1 where $\varepsilon_{ii} = 1, \varepsilon_{ij} = \varepsilon_{ji} = \pm 1$. Then $f = x_1^2 + \cdots + x_n^2$ is a homogeneous regular central element in S, so $A = S/(f)$ is an example of a homogeneous coordinate ring of a noncommutative quadric hypersurface in the sense of [2]. In this talk, we introduce graphical methods to compute $\underline{\mathrm{CM}}^{\mathbb{Z}}(S/(f))$. To do this, we associate each (± 1)-skew polynomial algebra S with a certain graph G. We present the four operations, called mutation, relative mutation, Knörrer reduction, and two points reduction for G, and show that they are powerful in computing $\underline{\mathrm{CM}}^{\mathbb{Z}}(S/(f))$. In fact, by using these four graphical methods, we can completely compute $\underline{\mathrm{CM}}^{\mathbb{Z}}(S/(f))$ up to $n \leq 6$. As a result, in the case $n \leq 6$, we see $\underline{\mathrm{CM}}^{\mathbb{Z}}(S/(f))$ is equivalent to one of $\mathcal{D}^b(\operatorname{mod} k^{2^i})$ where $0 \leq i \leq 5$. Moreover we also see that if $n \leq 6$, then the structure of $\underline{\mathrm{CM}}^{\mathbb{Z}}(S/(f))$ is determined by the number of irreducible components of the point scheme of S that are isomorphic to $\mathbb{P}^1$. (From this it follows that the conjecture proposed in [3] holds true for $n \leq 6$.)

This talk is based on the results of [1].

References

1. I. Mori and K. Ueyama, *Noncommutative Knörrer's periodicity theorem and noncommutative quadric hypersurfaces*, preprint, `arXiv:1905.12266`.
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