Constructions of rejective chains
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Let $\mathcal{C}$ be a Krull–Schmidt category. In [2], a chain $\mathcal{C} = \mathcal{C}_0 \supset \mathcal{C}_1 \supset \cdots \supset \mathcal{C}_n = 0$ of subcategories of $\mathcal{C}$ is called a total right rejective chain if the following conditions hold:

(a) $\mathcal{C}_i$ is a right rejective subcategory of $\mathcal{C}$;
(b) the Jacobson radical of the factor category $\mathcal{C}_{i-1}/[\mathcal{C}_i]$ is zero.

In this talk, we give various examples of total right rejective chains. It is known that total right rejective chains are deeply related right-strongly quasi-hereditary algebras which are a special class of quasi-hereditary algebras introduced by Ringel [3].

**Proposition 1** ([4, Theorem 3.22]). Let $A$ be an artin algebra. Then $A$ is right-strongly quasi-hereditary if and only if the category $\text{proj} A$ has a total right rejective chain.

The following theorem is one of main results of this talk. One is a refinement of [1, Proposition 1.6], and the other is a refinement of [1, Proposition 2.3] and [5, Proposition 3.1].

**Theorem 2.** Let $A$ be an artin algebra. If $A$ is a locally hereditary algebra or a Nakayama algebra with heredity ideal, then the category $\text{proj} A$ admits a total right rejective chain. In particular, the following statements hold.

1. If $A$ is a locally hereditary algebra, then $A$ is right-strongly quasi-hereditary.
2. Let $A$ be a Nakayama algebra. Then $A$ is a right-strongly quasi-hereditary algebra if and only if there exists a heredity ideal of $A$.

Next, we study $\Delta$-good module category $\mathcal{F}(\Delta)$ using rejective chains. In [6], it is shown that if the category $\mathcal{F}(\Delta)$ over a quasi-hereditary algebra $A$ has an additive generator $M$, then the endomorphism algebra $\text{End}_A(M)$ is quasi-hereditary. Motivated by this result, we give the following proposition.

**Proposition 3.** Let $A$ be a quasi-hereditary algebra and $\mathcal{F}(\Delta)$ the $\Delta$-good module category. Assume that $\mathcal{F}(\Delta)$ has an additive generator $M$ and multiplicity-free. Then the category $\mathcal{F}(\Delta)$ admits a total right rejective chain. In particular, the endomorphism algebra $\text{End}_A(M)$ is a right-strongly quasi-hereditary algebra.

**References**


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