

# POSITIVELY GRADED RINGS ARE MAXIMAL ORDERS AND GENERALIZED DEDEKIND RINGS

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Let  $R = \bigoplus_{n \in \mathbb{Z}_0} R_n$  be a positively graded ring which is a sub-ring of strongly graded ring of type  $\mathbb{Z}$ , where  $R_0$  is a Noetherian prime rings. We define a concept of  $\mathbb{Z}_0$ -invariant maximal order and show that  $R$  is a maximal order if and only if  $R_0$  is a  $\mathbb{Z}_0$ -invariant maximal order. If  $R$  is a maximal order, then we completely describe all  $v$ -invertible ideals. As an application, we show that  $R$  is a generalized Dedekind prime if and only if  $R_0$  is a  $\mathbb{Z}_0$ -invariant generalized Dedekind prime rings. We give example of  $\mathbb{Z}_0$ -invariant generalized Dedekind prime rings but neither generalized Dedekind prime rings nor maximal orders.

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