

# ON THE RADIUS OF THE CATEGORY OF EXTENSIONS OF MATRIX FACTORIZATIONS

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## 1. INTRODUCTION

Rouquier [5] has introduced the notion of the dimension of a triangulated category. As an analogue for abelian categories, Dao and Takahashi [2, 3] have introduced the notions of the dimension and radius of a full subcategory of an abelian category with enough projective objects. Our purpose of this talk is studying the dimension and radius of a full subcategory of the category of finitely generated modules over a commutative noetherian ring, and the dimension of the singularity category of a commutative noetherian ring.

For a noetherian ring  $R$  we denote by  $D_{\text{sg}}(R)$  the singularity category of  $R$ , i.e., the Verdier quotient of the bounded derived category of  $\text{mod } R$  by perfect complexes. Our main result yields the following corollary, which gives rise to an inequality of the dimensions of the singularity categories of 1-dimensional hypersurfaces. This corollary refines a recent result of Kawasaki, Nakamura and Shimada [4, Theorem 4.5], which assumes that the elements  $x_1, \dots, x_n$  are powers of distinct prime elements and that the local ring  $S$  is complete.

**Corollary 1.** *Let  $S$  be a regular local ring of dimension two and  $x_1, \dots, x_n \in S$ . Then one has*

$$\dim D_{\text{sg}}(S/(x_1 \cdots x_n)) \leq \sup_{1 \leq i \leq n} \{\dim D_{\text{sg}}(S/(x_i))\} + 1.$$

*In particular, if  $S/(x_i)$  has finite CM-representation type for  $1 \leq i \leq n$ , then*

$$\dim D_{\text{sg}}(S/(x_1 \cdots x_n)) \leq 1.$$

## REFERENCES

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