

Noetherian-like properties in polynomial and power series rings

Mi Hee Park

Chung-Ang University, Seoul, Korea

Email: mhpark@cau.ac.kr

There are several Noetherian-like properties, e.g., Noetherian spectrum, Laskerian, strong-finite-type (SFT), piecewise Noetherian property. We investigate the stability of such properties under polynomial and power series extensions. In particular, we show that for a nonzero SFT prime ideal P of a Prüfer domain D , the following statements are equivalent: (1) $D[[X]]_{P[[X]]}$ is Noetherian; (2) $\text{ht } P = 1$ and $\bar{k}[[X]] = \bar{D}[[X]]_{\bar{D} \setminus (0)}$, where $\bar{D} = D/P$ and \bar{k} is the quotient field of \bar{D} ; (3) $D[[X]]_{P[[X]]}$ is a valuation domain. As a corollary, we also show that for a Prüfer domain D , $D[[X]]$ is piecewise Noetherian if and only if D is Noetherian.

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