A Poisson \( \mathbb{C} \)-algebra \( R \) appears in classical mechanical system and its quantized algebra appearing in quantum mechanical system is a \( \mathbb{C}[[\hbar]] \)-algebra \( Q = R[[\hbar]] \) with star product \( * \) such that for any \( a, b \in R \subseteq Q \),

\[
a * b = ab + B_1(a,b)\hbar + B_2(a,b)\hbar^2 + \ldots
\]

subject to

\[
\{a,b\} = \hbar^{-1}(a * b - b * a)|_{\hbar=0}, \quad \cdots \quad (**)\]

where \( B_i : R \times R \rightarrow R \) are bilinear products. The given Poisson algebra \( R \) is recovered from its quantized algebra \( Q \) by \( R = Q/\hbar Q \) with Poisson bracket (**) which is called its semiclassical limit. But it seems that the star product in \( Q \) is complicate and that \( Q \) is difficult to understand at an algebraic point of view since it is too big. For instance, if \( \lambda \) is a nonzero element of \( \mathbb{C} \) then \( \hbar - \lambda \) is a unit in \( Q \) and thus a so-called deformation of \( R \), \( Q/(\hbar - \lambda)Q \), is trivial. Hence it seems that we need an appropriate \( \mathbb{F} \)-subalgebra \( A \) of \( Q \) such that \( A \) contains all generators of \( Q \), \( \hbar \in A \) and \( A \) is understandable at an algebraic point of view, where \( \mathbb{F} \) is a subring of \( \mathbb{C}[[\hbar]] \).

Here we discuss how to find nontrivial deformations from quantized algebras and how similar quantized algebras are to their semiclassical limits. Results are illustrated by examples.

**References**


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