Relationships between quantized algebras and their semiclassical limits

Sei-Qwon Oh

Chungnam National University

Email: sqoh@cnu.ac.kr

A Poisson $\mathbb{C}$-algebra R appears in classical mechanical system and its quantized algebra appearing in quantum mechanical system is a $\mathbb{C}[[\hbar]]$-algebra $Q = R[[\hbar]]$ with star product $*$ such that for any $a, b \in R \subseteq Q$,

$$a * b = ab + B_1(a,b)\hbar + B_2(a,b)\hbar^2 + \dots$$

subject to

$$\{a, b\} = \hbar^{-1}(a * b - b * a)|_{\hbar=0}, \qquad \cdots \quad (**)$$

where $B_i : R \times R \longrightarrow R$ are bilinear products. The given Poisson algebra R is recovered from its quantized algebra Q by $R = Q/\hbar Q$ with Poisson bracket $(**)$, which is called its semiclassical limit. But it seems that the star product in Q is complicate and that Q is difficult to understand at an algebraic point of view since it is too big. For instance, if λ is a nonzero element of $\mathbb{C}$ then $\hbar - \lambda$ is a unit in Q and thus a so-called deformation of R, $Q/(\hbar - \lambda)Q$, is trivial. Hence it seems that we need an appropriate $\mathbb{F}$-subalgebra A of Q such that A contains all generators of Q, $\hbar \in A$ and A is understandable at an algebraic point of view, where $\mathbb{F}$ is a subring of $\mathbb{C}[[\hbar]]$.

Here we discuss how to find nontrivial deformations from quantized algebras and how similar quantized algebras are to their semiclassical limits. Results are illustrated by examples.

References

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