The chain conditions on ideals in composite generalized power series rings

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Let R be a commutative ring with identity and $(\Gamma, \leq)$ a strictly ordered monoid. Let R be a commutative ring with identity and $(\Gamma, \leq)$ a strictly ordered monoid. We denote by $[\![R^{\Gamma,\leq}]\!]$ the set of all mappings $f : \Gamma \to R$ such that $\mathrm{supp}(f) := \{\alpha \in \Gamma \mid f(\alpha) \neq 0\}$ is an artinian and narrow subset of Γ. With pointwise addition, $[\![R^{\Gamma,\leq}]\!]$ is an (additive) abelian group. Moreover, for every $\alpha \in \Gamma$ and $f, g \in [\![R^{\Gamma,\leq}]\!]$, the set $X_\alpha(f, g) := \{(\beta, \gamma) \in \Gamma \times \Gamma \mid \alpha = \beta + \gamma, f(\beta) \neq 0, \text{ and } g(\gamma) \neq 0\}$ is finite; so this allows to define the operation of *convolution*:

$$(fg)(\alpha) = \textstyle\sum_{(\beta,\gamma)\in X_\alpha(f,g)} f(\beta)g(\gamma).$$

Then $[\![R^{\Gamma,\leq}]\!]$ is a commutative ring (under these operations) with unit element $\mathbf{e}$, namely $\mathbf{e}(0) = 1$ and $\mathbf{e}(\alpha) = 0$ for all $\alpha \in \Gamma^*$, which is called the *ring of generalized power series* of Γ over R, which is first introduced by P.Ribenboim.

Let $D \subseteq E$ be an extension of commutative rings with identity, I a nonzero proper ideal of D, $(\Gamma, \leq)$ a strictly ordered monoid, and $\Gamma^* = \Gamma \setminus \{0\}$. Set $D + [\![E^{\Gamma^*,\leq}]\!] = \{f \in [\![E^{\Gamma,\leq}]\!] \mid f(0) \in D\}$ and $D + [\![I^{\Gamma^*,\leq}]\!] = \{f \in [\![D^{\Gamma,\leq}]\!] \mid f(\alpha) \in I \text{ for all } \alpha \in \Gamma^*\}$. Then $D \subsetneq D + [\![I^{\Gamma^*,\leq}]\!] \subsetneq [\![D^{\Gamma,\leq}]\!] \subseteq D + [\![E^{\Gamma^*,\leq}]\!] \subseteq [\![E^{\Gamma,\leq}]\!]$.

In this talk, we give some conditions for the rings $D + [\![E^{\Gamma^*,\leq}]\!]$ and $D + [\![I^{\Gamma^*,\leq}]\!]$ to satisfy the ascending chain condition on principal ideals (ACCP) or Noetherian.

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