Nilpotent polynomials with non-nilpotent coefficients

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It is well known that the coefficients of nilpotent polynomials over noncommutative rings generally are not all nilpotent. We show that this remains true even under extremely strong restrictions on the set of nilpotents in the coefficient ring. If R is a ring and its set of nilpotents, $\mathrm{Nil}(R)$, satisfies $\mathrm{Nil}(R)^2 = 0$, then in general $\mathrm{Nil}(R[x]) \not\subseteq \mathrm{Nil}(R)[x]$. This is proven by constructing an explicit polynomial example. The smallest possible degree of such a polynomial is seven. Related problems are raised.

References

1. Ramon Antoine, *Nilpotent elements and Armendariz rings*, J. Algebra **319** (2008), no. 8, 3128–3140. MR 2408310
2. Ramon Antoine, *Examples of Armendariz rings*, Comm. Algebra **38** (2010), no. 11, 4130–4143. MR 2764855
3. George M. Bergman, *The diamond lemma for ring theory*, Adv. in Math. **29** (1978), no. 2, 178–218. MR 506890
4. Victor Camillo and Pace P. Nielsen, *McCoy rings and zero-divisors*, J. Pure Appl. Algebra **212** (2008), no. 3, 599–615. MR 2365335
5. Kwang Jin Choi, Tai Keun Kwak, and Yang Lee, *Rings whose nilpotents form a multiplicative set*, Comm. Algebra **46** (2018), no. 8, 3229–3240. MR 3788991
6. Roger A. Horn and Charles R. Johnson, *Topics in matrix analysis*, Cambridge University Press, Cambridge, 1991. MR 1091716
7. Tai Keun Kwak and Yang Lee, *Rings over which coefficients of nilpotent polynomials are nilpotent*, Internat. J. Algebra Comput. **21** (2011), no. 5, 745–762. MR 2827201
8. T. Y. Lam, A. Leroy, and J. Matczuk, *Primeness, semiprimeness and prime radical of Ore extensions*, Comm. Algebra **25** (1997), no. 8, 2459–2506. MR 1459571
9. Vassilis G. Papanicolaou, *Generalized (of confluent) Vandermonde determinants*, Online notes accessed at `http://www.math.ntua.gr/ papanico/` (2019), 1–6.
10. Agata Smoktunowicz, *Polynomial rings over nil rings need not be nil*, J. Algebra **233** (2000), no. 2, 427–436. MR 1793911
11. Janez Šter, *Rings in which nilpotents form a subring*, Carpathian J. Math. **32** (2016), no. 2, 251–258. MR 3587893

2010 *Mathematics Subject Classification.* Primary 16N40. Secondary 16S10, 16S15, 16S36, 16U99.