Nilpotent polynomials with non-nilpotent coefficients
Pace P. Nielsen
Brigham Young University
Email: pace@math.byu.edu

It is well known that the coefficients of nilpotent polynomials over noncommutative rings generally are not all nilpotent. We show that this remains true even under extremely strong restrictions on the set of nilpotents in the coefficient ring. If \( R \) is a ring and its set of nilpotents, \( \text{Nil}(R) \), satisfies \( \text{Nil}(R)^2 = 0 \), then in general \( \text{Nil}(R[x]) \not\subseteq \text{Nil}(R)[x] \). This is proven by constructing an explicit polynomial example. The smallest possible degree of such a polynomial is seven. Related problems are raised.

REFERENCES

1. Ramon Antoine, Nilpotent elements and Armendariz rings, J. Algebra 319 (2008), no. 8, 3128–3140. MR 2408310

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