On a cubical generalization of preprojective algebras
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In this abstract $K$ denotes a field of char $K = 0$ and $Q$ denotes a finite acyclic quiver.
Recall that the preprojective algebra $\Pi(Q) = KQ/(\rho)$ is the path algebra $KQ$ of the double quiver $\overline{Q}$ of $Q$ with the mesh relation $\rho = \sum_{a \in Q_1} a\alpha^* - \alpha^*\alpha$. It is an important mathematical object having rich representation theory and plenty of applications. In this joint work with M. Herschend, we study a cubical generalization $\Lambda = \Lambda(Q) := K\overline{Q}/(a, \rho)$ where $\Lambda_1$ is the commutator. We note that our algebra $\Lambda$ is a special case of algebras $\Lambda_{\lambda, \mu}$ introduced by Etingof-Rains [4], which is a special case of algebras $\Lambda_{\Phi}$ introduced by Cachazo-Katz-Vafa [2]. However, our algebra $\Lambda$ of very special case has intriguing properties, among other things it provides the universal Auslander-Reiten triangle.

We may equip $\Lambda$ with a grading by setting $\deg \alpha = 0, \deg \alpha^* := 1$ for $\alpha \in Q_1$. $\Lambda_1$ We introduce an algebra to be $A = A(Q) := \begin{pmatrix} KQ & \Lambda_1 \\ 0 & KQ \end{pmatrix}$ where $\Lambda_1$ is the degree 1-part of $\Lambda$.

We note that Etingof-Latour-Rains [3] showed that if $Q$ is a ADE-quiver, then $A$ is symmetric. We summarize existing results on the algebras $\Lambda$ and $A$.

Theorem 1. (1) $\Lambda$ is finite dimensional if and only if $Q$ is an ADE-quiver if and only if $A$ is 2-representation finite algebra. Assume that this is the case. Then $\Lambda$ is a stably 3-Calabi-Yau symmetric algebra. Moreover we have an isomorphism $\Lambda \cong \bigoplus_{M \in \text{ind } KQ} M \otimes_K M$ of $KQ$-modules.

(2) $\Lambda$ is infinite dimensional if and only if $Q$ is not an ADE-quiver $A$ is 2-representation infinite algebra. Assume this is the case. Then $\Lambda$ is graded coherent and 3-Calabi-Yau.

(3) In any case, the 2-quasi-Veronese algebra of $\Lambda$ is isomorphic to the 3-preprojective algebra of $\Lambda$. 2-APR-tilting operations on $A$ are compatible with reflections of quiver $Q$.

Let $Q$ be an ADE-quiver, $\hat{Q}$ the extended one and $G < SL(2)$ the corresponding finite subgroup. Then $\Lambda(\hat{Q})$ is Morita equivalent to the skew group algebra $H \ast G$ where $H = K[x, y]/((x, [x, y]), [y, [x, y]])$ is the Heisenberg algebra in two variables. The fixed subalgebra $H^G$ is Gorenstein. Applying a result by Amiot-Iyama-Reiten [1], we obtain our version of algebraic McKay correspondences giving descriptions of the stable categories of CM-modules over $H^G$.

Theorem 2. We have the following two equivalences of triangulated categories: $\text{CM}^Z H^G \simeq \text{D}^b(A(Q)), \text{CM}^H G \simeq C_2(A(Q))$ where $C_2$ denotes the 2-cluster category.

REFERENCES

2. F. Cachazo, S. Katz and C. Vafa, Geometric Transition and $N = 1$ Quiver Theories.

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