The Auslander-Reiten conjecture for non-Gorenstein rings

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The purpose of this talk is to study the vanishing of cohomology. The Auslander-Reiten conjecture is one of the long-standing conjectures about the vanishing, that is, for a Noetherian ring R and a finitely generated R-module M, $\mathrm{Ext}_R^i(M, M \oplus R) = 0$ for all $i > 0$ implies that M is a projective R-module. In this talk, we focus on the Auslander-Reiten conjecture for the case where R is commutative. In that case, the following result is fundamental.

Fact 1. Suppose that R is a commutative Noetherian local ring. Let Q be an ideal of R generated by a regular sequence on R. Then the Auslander-Reiten conjecture holds for R if and only if it holds for R/Q.

Motivated by this result, we explore the Auslander-Reiten conjecture for R/Q^ℓ in connection with that for R, where ℓ is a positive integer. Let us note that Q^ℓ do not preserve some homological properties, for example, Gorensteinness. Therefore R/Q^ℓ gives a new class of rings which satisfy the Auslander-Reiten conjecture. As a result of this talk, we have an affirmative answer to this question for the case where R is Gorenstein and ℓ is bounded above by the number of minimal generators of Q. Furthermore, we have two applications of the result. To state the applications, let us recall some notations.

Definition 2. (1) **(determinantal ring)** Let $s \le t$ be positive integers and $A[\mathbf{X}] = A[X_{ij}]_{1\le i\le s, 1\le j\le t}$ a polynomial ring over a commutative ring A. Let $\mathbb{I}_s(\mathbf{X})$ denote the ideal of $A[\mathbf{X}]$ generated by the maximal minors of the matrix (X_{ij}). Then $A[\mathbf{X}]/\mathbb{I}_s(\mathbf{X})$ is called a *determinantal ring over A*.

(2) **(Ulrich ideal)** Let $(R, \mathfrak{m})$ be a Cohen-Macaulay local ring and I an $\mathfrak{m}$-primary ideal. Then I is an *Ulrich ideal* if

(a) I is not a parameter ideal, but $I^2 = \mathfrak{q}I$ for some parameter ideal $\mathfrak{q}$.

(b) I/I^2 is a free R/I-module.

With these notations, we have the following, which is a goal of this talk.

Theorem 3. *The following assertions are true.*

(1) *Suppose A is either a complete intersection or a Gorenstein normal domain. Then the Auslander-Reiten conjecture holds for the determinantal ring $A[\mathbf{X}]/\mathbb{I}_s(\mathbf{X})$ if $2s \le t+1$.*

(2) *Let R be a Cohen-Macaulay local ring. If there is an Ulrich ideal such that R/I is a complete intersection, then the Auslander-Reiten conjecture holds for R.*

References

1. M. Auslander, I. Reiten, *On a generalized version of the Nakayama conjecture*, Proceedings of the American Mathematical Society **137** (2009), 1941–1944.
2. S. Kumashiro, *Auslander-Reiten conjecture for non-Gorenstein Cohen-Macaulay rings*, arXiv:1906.02669.

2010 *Mathematics Subject Classification.* 13C40, 13D07, 13H10.