The Auslander-Reiten conjecture for non-Gorenstein rings

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The purpose of this talk is to study the vanishing of cohomology. The Auslander-Reiten conjecture is one of the long-standing conjectures about the vanishing, that is, for a Noetherian ring \( R \) and a finitely generated \( R \)-module \( M \), \( \operatorname{Ext}^i_R(M, M \oplus R) = 0 \) for all \( i > 0 \) implies that \( M \) is a projective \( R \)-module. In this talk, we focus on the Auslander-Reiten conjecture for the case where \( R \) is commutative. In that case, the following result is fundamental.

**Fact 1.** Suppose that \( R \) is a commutative Noetherian local ring. Let \( Q \) be an ideal of \( R \) generated by a regular sequence on \( R \). Then the Auslander-Reiten conjecture holds for \( R \) if and only if it holds for \( R/Q \).

Motivated by this result, we explore the Auslander-Reiten conjecture for \( R/Q^\ell \) in connection with that for \( R \), where \( \ell \) is a positive integer. Let us note that \( Q^\ell \) do not preserve some homological properties, for example, Gorensteinness. Therefore \( R/Q^\ell \) gives a new class of rings which satisfy the Auslander-Reiten conjecture. As a result of this talk, we have an affirmative answer to this question for the case where \( R \) is Gorenstein and \( \ell \) is bounded above by the number of minimal generators of \( Q \). Furthermore, we have two applications of the result. To state the applications, let us recall some notations.

**Definition 2.**

1. **(determinantal ring)** Let \( s \leq t \) be positive integers and \( A[X] = A[X_{ij}]_{1 \leq i \leq s, 1 \leq j \leq t} \) a polynomial ring over a commutative ring \( A \). Let \( \mathbb{I}_s(X) \) denote the ideal of \( A[X] \) generated by the maximal minors of the matrix \( (X_{ij}) \). Then \( A[X]/\mathbb{I}_s(X) \) is called a determinantal ring over \( A \).

2. **(Ulrich ideal)** Let \((R, \mathfrak{m})\) be a Cohen-Macaulay local ring and \( I \) an \( \mathfrak{m} \)-primary ideal. Then \( I \) is an Ulrich ideal if
   - (a) \( I \) is not a parameter ideal, but \( I^2 = qI \) for some parameter ideal \( q \).
   - (b) \( I/I^2 \) is a free \( R/I \)-module.

With these notations, we have the following, which is a goal of this talk.

**Theorem 3.** The following assertions are true.

1. Suppose \( A \) is either a complete intersection or a Gorenstein normal domain. Then the Auslander-Reiten conjecture holds for the determinantal ring \( A[X]/\mathbb{I}_s(X) \) if \( 2s \leq t + 1 \).
2. Let \( R \) be a Cohen-Macaulay local ring. If there is an Ulrich ideal such that \( R/I \) is a complete intersection, then the Auslander-Reiten conjecture holds for \( R \).

**References**


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