A characterization of local rings of countable representation type

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All the contents of this article are taken from joint work with Justin Lyle and Ryo Takahashi [3]. We refer the reader to it for the details.

Cohen–Macaulay representation theory has been studied widely and deeply for more than four decades. Buchweitz, Greuel and Schreyer [2] proved that the local hypersurfaces of finite (resp. countable) CM-representation type,(that is, Cohen–Macaulay local rings possessing finitely/infinitely-but-countably many nonisomorphic indecomposable maximal Cohen–Macaulay modules) are precisely the local hypersurfaces of type (A_n) with $n \geq 1$, (D_n) with $n \geq 4$, and (E_n) with $n = 6, 7, 8$ (resp. (A_∞) and (D_∞)).

In this talk, we introduce another representation type, namely, *finite* CM_+*-representation type*. We say that a Cohen–Macaulay local ring has finite CM_+-representation type if there exist only finitely many isomorphism classes of indecomposable maximal Cohen–Macaulay modules that are *not* locally free on the punctured spectrum. Then, Araya, Iima and Takahashi [1] observed that the local hypersurfaces of type (A_∞) and (D_∞) has finite CM_+-representation type. Thus, it is natural to ask the following question.

Conjecture 1. *Let R be a complete local Gorenstein ring of dimension d not having an isolated singularity. Then the following two conditions are equivalent.*

(1) *The ring R has finite CM_+-representation type.*
(2) *There exist a complete regular local ring S and a regular system of parameters $x_0, \dots, x_d$ such that R is isomorphic to*

$$S/(x_0^2 + x_2^2 + \cdots + x_d^2) \quad \text{or} \quad S/(x_0^2 x_1 + x_2^2 + \cdots + x_d^2).$$

We give a complete answer to this conjecture in dimension one.

Theorem 2. *Let R be a homomorphic image of a regular local ring. Suppose that R does not have an isolated singularity but is Gorenstein. If $\dim R = 1$, the following are equivalent.*

(1) *The ring R has finite CM_+-representation type.*
(2) *There exist a regular local ring S and a regular system of parameters x, y such that R is isomorphic to $S/(x^2)$ or $S/(x^2 y)$.*

When either of these two conditions holds, the ring R has countable CM-representation type.

References

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2010 *Mathematics Subject Classification.* 13C60, 13H10, 16G60.