A new semistar operation on a commutative ring and its applications

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In this talk, a new semistar operation, called the \( q \)-operation, on a commutative ring \( R \) is introduced in terms of the ring \( Q_0(R) \) of finite fractions. It is defined as the map
\[
q : F_q(R) \rightarrow F_q(R) \text{ by } A \mapsto A_q := \{ x \in Q_0(R) \mid \text{there exists some finitely generated semiregular ideal } J \text{ of } R \text{ such that } Jx \subseteq A \} \text{ for any } A \in F_q(R),
\]
where \( F_q(R) \) denotes the set of nonzero \( R \)-submodules of \( Q_0(R) \). The main superiority of this semistar operation is that it can also act on \( R \)-modules. And we can also get a new hereditary torsion theory \( \tau_q \) induced by a (Gabriel) topology \( \{ I \mid I \text{ is an ideal of } R \text{ with } I_q = R_q \} \). Based on the existing literature of \( \tau_q \)-Noetherian rings by Golan and Bland et al., in terms of the \( q \)-operation, we can study them in more detailed and deep module-theoretic point of view, such as \( \tau_q \)-analogue of the Hilbert basis theorem, Krull’s principal ideal theorem, Cartan-Eilenberg-Bass theorem, and Krull intersection theorem.

References