A new semistar operation on a commutative ring and its applications

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In this talk, a new semistar operation, called the q-operation, on a commutative ring R is introduced in terms of the ring $Q_0(R)$ of finite fractions. It is defined as the map $q : \mathcal{F}_q(R) \to \mathcal{F}_q(R)$ by $A \mapsto A_q := \{x \in Q_0(R) \mid$ there exists some finitely generated semiregular ideal J of R such that $Jx \subseteq A\}$ for any $A \in \mathcal{F}_q(R)$, where $\mathcal{F}_q(R)$ denotes the set of nonzero R-submodules of $Q_0(R)$. The main superiority of this semistar operation is that it can also act on R-modules. And we can also get a new hereditary torsion theory τ_q induced by a (Gabriel) topology $\{I \mid I$ is an ideal of R with $I_q = R_q\}$. Based on the existing literature of τ_q-Noetherian rings by Golan and Bland *et al.*, in terms of the q-operation, we can study them in more detailed and deep module-theoretic point of view, such as τ_q-analogue of the Hilbert basis theorem, Krull's principal ideal theorem, Cartan-Eilenberg-Bass theorem, and Krull intersection theorem.

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