The characteristic variety of an elliptic algebra

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This is based on joint work with Alex Chirvasitu and S. Paul Smith [2, 3, 4].

There have been several attempts to define the space associated to a noncommutative ring. For a graded algebra over a field k, one established approach is to look at QGr A, the category of graded A-modules modulo the full subcategory consisting of torsion modules. When the algebra A is commutative and finitely generated in degree one, the category QGr A is equivalent to the category of quasi-coherent sheaves on Proj A. Thus, for a noncommutative algebra A, we may consider QGr A as the category of “quasi-coherent sheaves” on the associated “noncommutative projective scheme”.

To understand QGr A, the first things one should look at are objects coming from point modules:

**Definition 1.** Let A be a nonnegatively graded k-algebra that is finitely generated in degree one. A graded A-module M is called a point module if it is cyclic and satisfies

\[ \dim_k M_i = \begin{cases} 
1 & \text{if } i \geq 0, \\
0 & \text{if } i < 0. 
\end{cases} \]

Artin-Tate-Van den Bergh [1] showed that the point modules are parametrized by a space called the point scheme, which is defined as an inverse limit of schemes. Each point module defines a simple object in QGr A. Point modules have played a crucial role in the study of Artin-Schelter regular algebras.

In 1989, Feigin and Odesskii introduced a family of algebras \( Q_{n,k}(E,\tau) \) parametrized by an elliptic curve \( E \) over \( \mathbb{C} \), a closed point \( \tau \in E \), and coprime integers \( n > k \geq 1 \). This is a huge generalization of higher dimensional Sklyanin algebras, and provides flat deformations of polynomial algebras when \( \tau \) varies.

The aim of this talk is to describe the major component of the point scheme of the elliptic algebra \( Q_{n,k}(E,\tau) \), which we call the characteristic variety. For a higher dimensional Sklyanin algebra, the characteristic variety is the elliptic curve \( E \) and it is the only non-discrete irreducible component of the point scheme. For other elliptic algebras, the characteristic variety depends on the negative continued fraction of the rational number \( n/k \) and is realized as the quotient of a product of copies of \( E \) by a finite group.

**References**


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