Hochschild cohomology of Beilinson algebras of graded down-up algebras

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Let \( k \) be an algebraically closed field of char \( k = 0 \). A graded \( k \)-algebra \( A(\alpha, \beta) := k(x, y)/(x^2y - \beta yx^2 - \alpha xy, xy^2 - \beta y^2x - \alpha yxy) \), \( \deg x = m, \deg y = n \in \mathbb{N}^+ \) with parameters \( \alpha, \beta \in \mathbb{K} \) is called a graded down-up algebra. It is known that a graded down-up algebra \( A = (\alpha, \beta) \) is a noetherian AS-regular algebra of dimension 3 if and only if \( \beta \neq 0 \) ([4]). By the special case of [5, Theorem 4.14], if \( A = A(\alpha, \beta) \) is a graded down-up algebra with \( \beta \neq 0 \), then the Beilinson algebra \( \nabla A \) of \( A \) is extremely Fano of global dimension 2, and there exists an equivalence of triangulated categories \( \mathbb{D}^b(\text{tails } A) \cong \mathbb{D}^b(\text{mod } \nabla A) \), where \( \text{tails } A \) is the noncommutative projective scheme of \( A \) in the sense of [1].

The aim of our talk is to investigate the Hochschild cohomology groups \( \text{HH}^i(\nabla A) \) of \( \nabla A \) of a graded down-up algebra \( A = A(\alpha, \beta) \) with \( \beta \neq 0 \). If \( \deg x = \deg y = 1 \), then a description of the Hochschild cohomology group \( \text{HH}^i(\nabla A) \) of \( \nabla A \) has been obtained using a geometric technique ([2, Table 2]). In this talk, for any \( \deg x = 1, \deg y = n \geq 2 \), we give the dimension formula of \( \text{HH}^i(\nabla A) \) for each \( i \geq 0 \). In this case, the Beilinson algebra \( \nabla A \) of \( A \) is given by the following quiver \( Q \) with relations \( f_i = 0 (1 \leq i \leq n), g = 0 \):

\[
Q := \begin{array}{ccccccccccc}
x_1 & \rightarrow & x_2 & \cdots & x_{n-1} & \rightarrow & x_n & \rightarrow & n+1 & \rightarrow & n+2 & \cdots & 2n+2, \\
y_1 & & y_2 & & y_n & & y_{n+1} & & y_{n+2} & & & &
\end{array}
\]

\[
f_i := x_i x_{i+1} y_{i+2} - \beta y_i x_i n x_i + n+1 - \alpha x_i y_i n x_i+1, \\
g := x_1 y_2 y_{n+2} - \beta y_1 y_{n+1} x_{2n+1} - \alpha y_1 y_{n+1} y_{n+2}.
\]

In particular, it turns out from our dimension formula that the group structure of \( \text{HH}^i(\nabla A) \) depends on the values of \( \alpha^2 + 4 \beta \) and \( \delta_n := (1, 0, 0) \left( \begin{smallmatrix} 0 & 1 \\ \beta & 0 \end{smallmatrix} \right)^n (0) \) ([3, Theorem 1.4]). Using the fact that Hochschild cohomology is invariant under derived equivalence, our result implies the following: Let \( A = A(\alpha, \beta) \) and \( A' = A(\alpha', \beta') \) be graded down-up algebras with \( \deg x = 1, \deg y = n \geq 1 \), where \( \beta \neq 0, \beta' \neq 0 \). If \( \delta_n := (1, 0, 0) \left( \begin{smallmatrix} 0 & 1 \\ \beta & 0 \end{smallmatrix} \right)^n (0) = 0 \) and \( \delta'_n := (1, 0, 0) \left( \begin{smallmatrix} 0 & 1 \\ \beta' & 0 \end{smallmatrix} \right)^n (0) \neq 0 \), then \( \mathbb{D}^b(\text{tails } A) \not\cong \mathbb{D}^b(\text{tails } A') \) ([3, Corollary 1.5]).

REFERENCES


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