

A remark on graded countable Cohen-Macaulay representation type

Naoya Hiramatsu

National Institute of Technology, Kure College

Email: hiramatsu@kure-nct.ac.jp

In the representation theory of Cohen-Macaulay algebras, a classifying the algebras with respect to the complexity of the classification of maximal Cohen-Macaulay modules over them is a major subject. It has been conjectured that all Cohen-Macaulay algebras split into three classes, that is, *Cohen-Macaulay discrete* (including *finite*), *Cohen-Macaulay tame* and *Cohen-Macaulay wild*. For graded cases, the studies are investigated by Eisenbud and Herzog [4], Stone [5], Drozd and Tovpyha [3] and so on. In this talk, we shall give a remark on graded countable Cohen-Macaulay representation type.

Let $R = \bigoplus_{i=0}^{\infty} R_i$ be a commutative positively graded ring with $R_0 = k$ an algebraically closed field. Let S be a graded Noetherian normalization. That is, S is a graded polynomial subring of R such that R is a finitely generated graded S -module. A finitely generated graded R -module M is said to be maximal Cohen-Macaulay (MCM) if M is graded free as a graded S -module. We say that a graded Cohen-Macaulay ring R is of graded countable CM representation type if there are infinitely but only countably many isomorphism classes of indecomposable graded Cohen-Macaulay R -modules up to shift.

Theorem 1. *Let R be of graded countable CM representation type. For each graded free S -module F there are finitely many isomorphism classes of MCM R -modules which are isomorphic to F as graded S -modules.*

To prove the theorem we consider the analogy of a module variety for finitely generated modules over a finite dimensional algebra, which was introduced by Dao and Shipman [2].

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