On Representation of a Ring with unity on a Module over a Ring with Unity

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The representation of rings on finite dimension vector spaces has been generalized to the representation of rings on modules over a commutative ring. Let S be a commutative ring with unity and M an S-module. A representation of ring R with unity on an S-module M is a ring homomorphism from R to the ring of endomorphisms of M. An S-module associated with a representation of R is called a representation module of R. For any ring homomorphism $f\colon R \to S$, we define a representation of ring R with unity on M via f, and it is called an f-representation of ring R which is a special case of the representation of ring R on an S-module. This S-module associated with the f-representation of ring R is called an f-representation module of R.

The result of our study is generalized Schur's Lemma. If an S-module is a representation module of ring R then it is is an R-S-bimodule, and every R-S-bimodule is a representation module of R. However, a bimodule is not necessarily an f-representation module of the ring. Furthermore, for two ring homomorphisms f, g from R to S, we obtained a sufficient condition of the equivalent of an f-representation and a g-representation. We also find some a sufficient condition of a module homomorphism becomes a morphism from an f-representation to a g-representation. This study also reveals that the sufficient condition of f-representation of the ring R on a finite dimension free module over a principal ideal domain S is decomposable and completely reducible.

In the case of S not commutative, we give the sufficient condition of the S-module M becomes the representation module of R. The category of f-representation modules of ring R is Abelian and Morita equivalent to the category of modules over an R-algebra. Thus, if the category of modules over the R-algebra which is equivalent to the category of f-representation modules of R satisfies the Krull-Schmidt Theorem, then the category of f-representation modules of R also satisfies Krull-Schmidt's Theorem.

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