Recollement of comodule categories over coalgebra objects

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One approach [3] to categorify representation theory is to replace an algebra by certain nice "additive 2-category" $\mathcal{A}$, and finite dimensional modules by abelian categories (with finitely many simples and consists only of finite length objects) that are equipped with an action of $\mathcal{A}$.

It turns out that the categorified version of an short exact sequence is equivalent to specifying a recollement $(\mathbf{L}, \mathbf{M}, \mathbf{N})$ of abelian categories.

It is well-known that if $\mathbf{M}$ in a recollement of abelian categories (as shown above) is a module category, say $\mathrm{mod}(A)$, of an algebra A, then there will be an idempotent e of A so that $\mathbf{L} \simeq \mathrm{mod}(A/AeA)$ and $\mathbf{N} \simeq \mathrm{mod}(eAe)$.

However, unlike the special case of $\mathcal{A}$ being a tensor category where $\mathcal{A}$-modules can take the form of module categories [1], we can only guarantee an $\mathcal{A}$-module takes the form of a comodule category $\mathrm{comod}_{\mathcal{A}}(\mathrm{C})$ over a coalgebra object C in the collection of morphism categories of $\mathcal{A}$ [2]. In this talk, we explain the analogue of the characterisation of recollements of module categories in this more general setting.

This is a joint work with Vanessa Miemietz (arXiv: 1901.04685).

References

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