ON TWO SIDED HARADA RINGS AND QF RINGS

Yoshitomo Baba

Osaka Kyoiku University

Email: ybaba@cc.osaka-kyoiku.ac.jp

Let R be a basic artinian ring, let $\{e_i\}_{i=1}^n$ be a complete set of orthogonal primitive idempotents of R and let $\{f_i\}_{i=1}^k \subseteq \{e_i\}_{i=1}^n$. A sequence $f_1 R$, $f_2 R$, ..., $f_k R$ is called a *right co-H-sequence* of R if the following (CHS1), (CHS2), (CHS3) hold:

(CHS1) For each $i = 1, 2, \ldots, k-1$, there exists an R-isomorphism $\xi_i : f_i R_R \to f_{i+1} J_R$.

(CHS2) The last term $f_k R_R$ is injective.

(CHS3) $f_1 R$, $f_2 R$, ..., $f_k R$ is the longest sequence among the sequences which satisfy (CHS1),(CHS2), i.e., there does not exist an R-isomorphism: $f R_R \to f_1 J_R$, where $f \in \{e_i\}_{i=1}^n$.

And, we call an artinian ring R *a left Harada ring* if there exists a basic set $\{e_{i,j}\}_{i=1,j=1}^{m\ n(i)}$ of orthogonal primitive idempotents of R' such that $e_{i,n(i)}R$, $e_{i,n(i)-1}R$, ..., $e_{i,1}R$ is a right co-H-sequence of R for all $i = 1, 2, \ldots, m$.

Left Harada ring is first studied by M. Harada in [1]. K. Oshiro further studied it and called the ring a left Harada ring (abbreviated left H-ring) in [2]. Many results on one sided Harada rings are given in [3].

In this talk, using a new concept "weak co-H-sequence", we characterize two sided Harada rings and give the relationship between two sided Harada rings and QF rings.

References

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2. K. Oshiro, *Lifting modules, extending modules and their applications to QF-rings*, Hokkaido Math. J. **13** (1984), 310–338.
3. Y. Baba and K. Oshiro, *Classical artinian rings and related topics*, World Scientific (2009).