2-categorical Cohen-Montgomery duality between categories with I-pseudo-actions and I-graded categories for a small category I

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Throughout this talk $\Bbbk$ denotes a commutative ring. We first note that a group pseudo-action of a group G on a category $\mathcal{C}$ defined by Deligne [2] and Drinfeld–Gelaki–Nikshych–Ostrik [3] is nothing but a pseudofunctor from G as a groupoid with a single object $*$ to the 2-category **CAT** of categories sending $*$ to $\mathcal{C}$. Thus if $\mathcal{C}$ is a small $\Bbbk$-category, then it is just a pseudofunctor $X\colon G \to \Bbbk\text{-}\mathbf{Cat}$ with $X(*) = \mathcal{C}$, where $\Bbbk$-**Cat** is the 2-category of small $\Bbbk$-categories. We denote by G-**Cat** the 2-category of small $\Bbbk$-categories with G-pseudo-actions, and by G-**GrCat** the 2-category of small G-graded $\Bbbk$-categories. By generalizing the main result in [1] it is possible to show that a 2-functor $?/G\colon G\text{-}\mathbf{Cat} \to G\text{-}\mathbf{GrCat}$ defined by extending the orbit category construction is a 2-equivalence with a 2-quasi-inverse $?\#G\colon G\text{-}\mathbf{GrCat} \to G\text{-}\mathbf{Cat}$ defined by extending the smash product. By replacing the group G by a small category I we extend this result. Denote by $\mathrm{Pfun}(I, \Bbbk\text{-}\mathbf{Cat})$ the 2-category of pseudofunctors $I \to \Bbbk\text{-}\mathbf{Cat}$, and by I-**GrCat** the 2-category of small I-graded $\Bbbk$-categories. Then we can generalize the Grothendieck construction to a 2-functor $\int_I\colon \mathrm{Pfun}(I, \Bbbk\text{-}\mathbf{Cat}) \to I\text{-}\mathbf{GrCat}$ and define the smash product 2-functor $?\#I\colon I\text{-}\mathbf{GrCat} \to \mathrm{Pfun}(I, \Bbbk\text{-}\mathbf{Cat})$ in such a way that they are 2-quasi-inverses to each other. Of course, if $I = G$ then we have $\int_I = ?/G$ and $?\#I = ?\#G$.

References

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