g-polytopes of Brauer graph algebras
Toshitaka Aoki
Nagoya University
Email: m15001d@math.nagoya-u.ac.jp

Inspired by a work of Hille [2], Asashiba-Mizuno-Nakashima [1] studied simplicial complexes of two-term tilting complexes over finite dimensional symmetric algebras $A$. For $0 \leq j \leq n-1$, the set of $j$-dimensional faces consists of the set of $g$-vectors $\{gT_1, \ldots, gT_{j+1}\}$ for basic two-term pretilting complexes $T = \bigoplus_{i=1}^{j+1} T_i$ having $j+1$ indecomposable direct summands, where $n$ is the number of simple modules of $A$. The $g$-polytope $\Delta(A)$ of $A$ is given by $(n-1)$-dimensional faces

$$\Delta(A) := \bigcup_{T \in \text{2-dit}A} C_{\leq 1}(T) \subseteq \mathbb{R}^n,$$

where $C_{\leq 1}(T)$ is the convex hull of $n+1$ vectors $0, gT_1, \ldots, gT_n$ for a basic two-term tilting complex $T = \bigoplus_{i=1}^n T_i$. Note that the $g$-polytope can be regarded as a truncated version of $g$-vector cones since we have $C_{\leq 1}(T) = \{\sum_{i=1}^n a_i gT_i | 0 \leq a_i \leq 1 \text{ for all } i = 1 \ldots, n\}$.

Due to the result of [1], the convexity and symmetry of $g$-polytopes are quite interesting in tilting mutation theory. One of their aims is to introduce the $g$-polytope as a new derived invariant of Brauer tree algebras. Note that Brauer tree algebras are $\tau$-tilting-finite symmetric algebras, namely, having only finitely many isomorphism classes of basic two-term tilting complexes.

Theorem 1. [2] Let $G$ be a Brauer tree and $A_G$ the associated Brauer tree algebra. Then $\Delta(A_G)$ is convex and satisfies $\Delta(A_G) = -\Delta(A_G)$. Therefore, if two Brauer tree algebras $A_G$ and $A_{G'}$ are derived equivalent, then we have $\Delta(A_G) \cong \Delta(A_{G'})$.

An aim of this talk is to give a generalization for non-$\tau$-tilting-finite symmetric algebras. In this case, we mainly study the closure $\overline{\Delta}(A)$ rather than $\Delta(A)$ itself. Finally, we conclude that the closure of $g$-polytopes of Brauer graph algebras is invariant under iterated mutation.

Proposition 2. Let $A$ be a symmetric algebra. If any algebra $B$ obtained by iterated mutation from $A$ satisfies $\overline{\Delta}(B) = -\overline{\Delta}(B)$, then we have $\overline{\Delta}(A) \cong \overline{\Delta}(B)$.

Theorem 3. Let $G$ be a Brauer graph and $A_G$ the associated Brauer graph algebra. Then $\overline{\Delta}(A_G)$ is convex and satisfies $\overline{\Delta}(A_G) = -\overline{\Delta}(A_G)$. Therefore, if two Brauer graph algebras $A_G$ and $A_{G'}$ are obtained by iterated mutation each other, then we have $\overline{\Delta}(A_G) \cong \overline{\Delta}(A_{G'})$.

Furthermore, we determine all integral lattice points of $\overline{\Delta}(A_G)$. We use a geometric model of a classification of two-term tilting complexes over Brauer graph algebras established by Adachi-Aihara-Chan.

References