g-polytopes of Brauer graph algebras

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Inspired by a work of Hille [2], Asashiba-Mizuno-Nakashima [1] studied simplicial complexes of two-term tilting complexes over finite dimensional symmetric algebras A. For $0 \leq j \leq n-1$, the set of j-dimensional faces consists of the set of g-vectors $\{g^{T_1}, \ldots, g^{T_{j+1}}\}$ for basic two-term pretilting complexes $T = \bigoplus_{i=1}^{j+1} T_i$ having $j+1$ indecomposable direct summands, where n is the number of simple modules of A. The *g-polytope* $\Delta(A)$ of A is given by $(n-1)$-dimensional faces

$$\Delta(A) := \bigcup_{T \in \mathsf{2\text{-}tilt} A} C_{\leq 1}(T) \ \subseteq \mathbb{R}^n,$$

where $C_{\leq 1}(T)$ is the convex hull of $n+1$ vectors $0, g^{T_1}, \ldots, g^{T_n}$ for a basic two-term tilting complex $T = \bigoplus_{i=1}^{n} T_i$. Note that the g-polytope can be regarded as a truncated version of g-vector cones since we have $C_{\leq 1}(T) = \{\sum_{i=1}^{n} a_i g^{T_i} \mid 0 \leq a_i \leq 1 \text{ for all } i = 1 \ldots, n\}$.

Due to the result of [1], the convexity and symmetry of g-polytopes are quite interesting in tilting mutation theory. One of their aims is to introduce the g-polytope as a new derived invariant of Brauer tree algebras. Note that Brauer tree algebras are *τ-tilting-finite* symmetric algebras, namely, having only finitely many isomorphism classes of basic two-term tilting complexes.

Theorem 1. [2] *Let G be a Brauer tree and A_G the associated Brauer tree algebra. Then $\Delta(A_G)$ is convex and satisfies $\Delta(A_G) = -\Delta(A_G)$. Therefore, if two Brauer tree algebras A_G and $A_{G'}$ are derived equivalent, then we have $\Delta(A_G) \cong \Delta(A_{G'})$.*

An aim of this talk is to give a generalization for non-τ-tilting-finite symmetric algebras. In this case, we mainly study the closure $\overline{\Delta}(A)$ rather than $\Delta(A)$ itself. Finally, we conclude that the closure of g-polytopes of Brauer graph algebras is invariant under iterated mutation.

Proposition 2. *Let A be a symmetric algebra. If any algebra B obtained by iterated mutation from A satisfies $\overline{\Delta}(B) = -\overline{\Delta}(B)$, then we have $\overline{\Delta}(A) \cong \overline{\Delta}(B)$.*

Theorem 3. *Let G be a Brauer graph and A_G the associated Brauer graph algebra. Then $\overline{\Delta}(A_G)$ is convex and satisfies $\overline{\Delta}(A_G) = -\overline{\Delta}(A_G)$. Therefore, if two Brauer graph algebras A_G and $A_{G'}$ are obtained by iterated mutation each other, then we have $\overline{\Delta}(A_G) \cong \overline{\Delta}(A_{G'})$.*

Furthermore, we determine all integral lattice points of $\overline{\Delta}(A_G)$. We use a geometric model of a classification of two-term tilting complexes over Brauer graph algebras established by Adachi-Aihara-Chan.

References

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2. L. Hille, *Tilting modules over the path algebra of type $\mathbb{A}$, polytopes, and Catalan numbers*, Amer. Math. Soc., Providence, RI, **652** (2015), 91–101.