

# On balanced Auslander–Dlab–Ringel algebras

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Many algebras that appear in representation theory and algebraic geometry often fall into the following two classes at the same time. One is the class of Koszul algebras introduced by Priddy [6], and the other is the class of quasi-hereditary algebras introduced by Cline, Parshall, and Scott [3]. These two classes of algebras exhibit two different forms of dualities - the Koszul duality [2] and the Ringel duality [7]. One sufficient condition for the two dualities of a Koszul quasi-hereditary algebra to commute is given by Mazorchuk [5], and an algebra that satisfies such a condition is called a *balanced algebra* nowadays.

In ring theory, one interesting class of quasi-hereditary algebras is given by the *Auslander–Dlab–Ringel algebras*. The class was first studied by Auslander in [1] and subsequently by Dlab and Ringel in [4] to give a construction of algebras with finite global dimensions.

In this talk, we will explain in slightly more details about the aforementioned classes of algebras, and will give a sufficient condition for Auslander–Dlab–Ringel algebras to be balanced.

## REFERENCES

1. M. Auslander, *Representation dimension of Artin algebras*, Lecture notes, Queen Mary College, 1971.
2. A. Beilinson, V. Ginzburg, W. Soergel, *Koszul duality patterns in representation theory*, J. Amer. Math. Soc. **9** (1996), no. 2, 473–527.
3. E. Cline, B. J. Parshall, L. L. Scott, *Finite dimensional algebras and highest weight categories*, J. reine angew. Math. **391** (1988), 85–99.
4. V. Dlab, C. M. Ringel, *Every semiprimary ring is the endomorphism ring of a projective module over a quasi-hereditary ring*, Proc. Amer. Math. Soc. **107** (1989), no. 1, 1–5.
5. V. Mazorchuk, *Koszul duality for stratified algebras. I. Balanced quasi-hereditary algebras*, Manuscripta Math. **131** (2010), no. 1–2, 1–10.
6. S. B. Priddy, *Koszul Resolutions*, Trans. Amer. Math. Soc., **152** (1970), 39–60.
7. C. M. Ringel, *The category of modules with good filtrations over a quasi-hereditary algebra has almost split sequences*, Math. Z., **208** (1991), no. 2, 209–223.